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## Using Problem-Based Learning to Improve College Students' Mathematical Argumentation Skills

Bambang Aryan Soekisno<sup>1\*</sup>, Yaya S. Kusumah<sup>2</sup>, Jozua Sabandar<sup>2</sup>, Darhim<sup>2</sup>

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### Abstract

Problem-based learning are very common in mathematics teaching and learning in college level where students are exposed to problems and required to solve them. However, in general, when students are making efforts to solve the problems, they usually implement direct problem solving strategy without involving argumentation as one of the most important component of the problem solving process. Through argumentation, a reasoning process will take place and will involve thinking about what data gathered, what theorem support the process, how rebuttal is given, and what final claim to confirm. A student is able to comprehend a certain problem meaningfully when he/she is able to state the reasons, to elaborate the data, to express assurance, and even to make a claim of a problem in correct ways. Therefore, in order to see if the students have already acquired the skills to express mathematics problem meaningfully, they can be asked to express ideas orally and rewrite the ideas in mathematical arguments. Thus, this research was intended to find out if the students have improved their mathematical argumentation in Calculus 1 course. A learning process that facilitates the students to develop arguments is needed in order to improve their mathematical argumentation skills. Therefore, this research took students at the Dept. of Mathematics Education in UHAMKA University as the subjects in which four classes of them were taken as the samples using purposive random sampling technique. The four classes were then divided into two groups: two classes as control groups and two other classes as experimental groups. In this research, the teaching and learning process in the experimental groups used Problem-based Learning (PBM); meanwhile, conventional learning process (KS) was implemented in the control groups. There were 141 students involved in this research. And the instrument used in this research was a test intended to measure mathematical argumentation skills. The data was analyzed using *t*-test and ANOVA through one and two lines. Based on the analysis, this research found that there was significant differences in mathematical argumentation between PAM Groups (upper, middle, and lower) on Problem-based Learning. There was differences in the improvement of PAM for upper and middle groups. Significantly, the improvement of students' mathematical argumentation skills in PAM groups using Problem-based Learning is better than those using conventional learning. There was also significant improvement on students' mathematical argumentation skills in each of PAM groups between those using Problem-based Learning and Conventional Learning. Collectively, both factors of PM groups and learning approach have given significant effects on students' mathematical argumentation skills.

**Key words:** Mathematical argumentation skills, Problem-based learning

### Introduction

The ability to express ideas supported by data and based on sufficient theories of a mathematical problem, both oral and written, is an important part of mathematical abilities that students in college level need to have. Ideas that are supported by sufficient data and based on appropriate theories would provide students with sufficient understanding about mathematical concepts. Sufficient reasons support explanations about something to be considered as right or wrong. In addition, reasons would also support an interpretation of many concepts. Changes would happen when one changes their understanding on certain concepts and conceptual framework they use. It also may happen when one redesign the framework to accommodate new perspectives.

The ability to express mathematical problems can be seen when students convey their ideas orally and rewrite their ideas into mathematical arguments. Ideas about optimizing the ability to make arguments in mathematics

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such as expressing reasons, data, theories, writing, and developing discourses have become one of the alternatives of finding answers on certain problems. That is why, it is not impossible that a doer and a user of mathematics look for forms, models, and trick during working on mathematics problems.

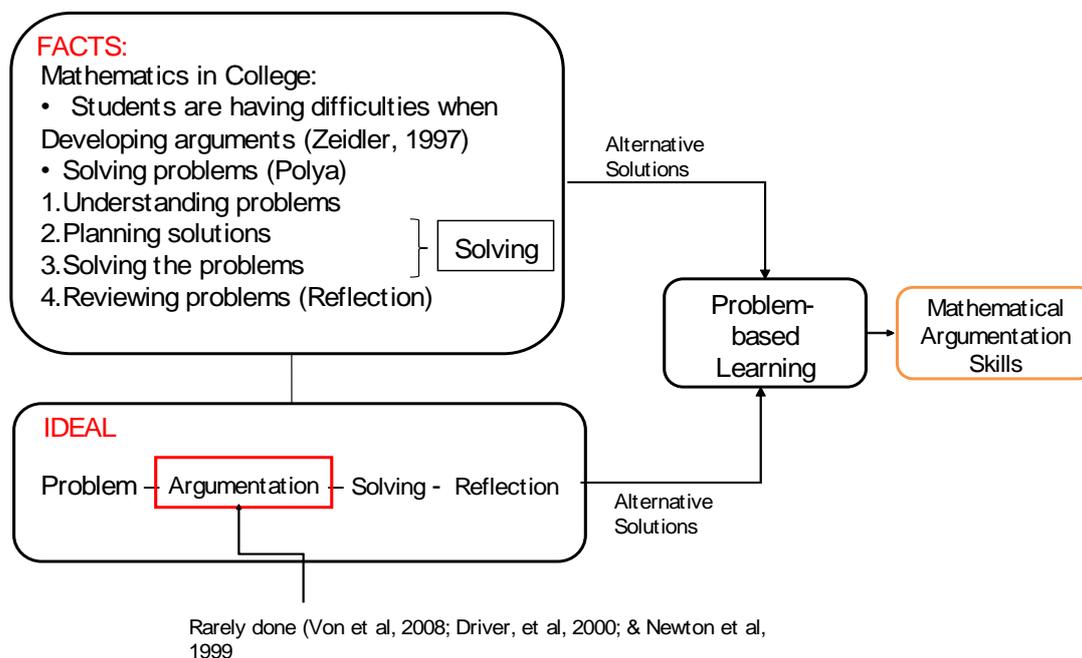
The process of searching for solutions of problems is surely not a simple reasoning process. It needs higher level of reasoning. The ability to collect information and data, to express arguments, to determine supporting theories, to decide the pathway of problem solving process, have become part of reasoning process that make students possible to solve the problems.

The ability to make an argumentation is an important part of mathematics learning. Therefore, students need to practice it as often as possible. They need to have argument skill to make them able to solve problem critically. Argumentation are the essence of scientific reasoning (Cross, 2007).

The ability is also a foundation of logical and critical thinking. According to Ennis (1981), critical thinking is an ability to express reasons based on what one beliefs. The argumentation ability involves skills to express a reasons critically supported by data and theories of mathematics problems (Logic).

Argumentation as a foundation of critical thinking and logic is still viewed as something difficult by students (Zeidler, 1997). Von et al. (2008), Driver, et al. (2000) and Newton, et al. (1999) said that the difficulties the students encounter when developing argumentation is caused by lack of pedagogical skills of lecturers in developing argumentation in classroom. Students often find difficulties when developing critical arguments due to poor learning process in classroom. However, in classroom, college students are often encountered with problems that they need to find the solution through argumentation process.

Before arriving to the solving problem phase, students need to go through a series of steps including reasoning process, data gathering, identifying theorem, rebuttals, and make some claims. Later, the problems can be solved with hopes that the process of solving problems are in the right directions.



Picture 1. Logical Framework for Improving Students' Mathematical Argumentation Skills

Argumentation skills are critical and logical thinking about the relationship between concepts and situations. The skills are very useful to explain about facts, procedures, concepts, and methods of solving problems that are interrelated one another. It is expected that the higher the students' mathematical argumentation skills, the better their ability to express reasons for certain solutions or answers.

Learning process needs to change in order to prepare students to face new situations. Students' skills in asking questions, searching and finding for appropriate resources to answers the questions, and communicating effectively the solutions that they have from others. Problem-based learning is an approach in mathematics

education that helps students develop critical and logical thinking and other skills needed in order to be able to communicate successfully.

Arguments are rational ways that one take to answer questions, to face issues, and to rebut and solve problems. An argument consists of a claim (Solution) that is supported by many principles (Assurance), proofs, and rebuttals that contradict with other potential arguments. Developing an argument in learning process can potentially solve problems.

Mathematical argumentation are important when one learns about how to solve many kind of big problems. It is also a powerful method that can be used to measure the ability in solving problems. The arguments are also useful to solve both poor-structured and well-structured problems (Jonassen, 2010). Moreover, Nussbaum & Sinatra (2003) said that students need to show that they are able to improve their reasoning over problems when they find solutions; or when developing an arguments in order to find appropriate answers scientifically.

There are many factors inhibit learning process such as students' lower reasoning level including argumentation. The most common weakness that the students have in making argumentation is in counter-argument. When a student is asked to make arguments to support and challenge something, he/she usually elaborate many reasons to support his/her positions (Stein & Bernas, 1999). Qualification and rebuttal are rarely used in analyzing arguments in mathematics education; and they are very useful in analyzing arguments made by students (Inglis et al, 2007).

There are many efforts have been done in many developed countries intended to find the causes and solutions to problem of lack mathematical argumentation skills of students. The efforts were done using many kinds of educational theories, learning models and approaches which resulted in improvement of knowledge about argument. Experts such as Conner (2008) provides a description about correlation between argument and proofs in geometry class. Meanwhile, Halpren (2003) identifies a process intended to analyze students' argument elaboration that involves reading and evaluating the argument based on the strength of correlation between premises, conclusions, assumptions, and counter-arguments.

Through problem based learning, students are expected to think critically, to analyze complex problems and real-life problems, to work cooperatively in small groups, to be skillful in making effective and accurate communication both verbal and non-verbal in order to show that they have mathematical arguments. Students' mathematical arguments will improve if they are involved in problem based learning, especially unstructured problems and interpretations over alternative solutions that needs argumentation. Students are required to remember information with minimum reasons in order to involve in making argumentation. Problem based learning environment usually exposes students to claims and alternative solutions that need to handle by them through argumentation.

From the perspective of pedagogy, the objective of learning for adults is to provide opportunities to students to doing math. Nowadays, learning focuses on the use of surrounding environment as the source such as in problem based learning. Problem based learning is a classroom activity intended to organize learning about problem solving. It is also about providing opportunities to students to express their arguments and mathematical ideas, and to communicate with their peers through interaction of components in classroom. This is in accordance with NCTM (2000) which says that developing learning environment that challenge and support important learning component is critical.

A learning process that provide many opportunities to students to develop arguments called as problem based learning. The learning begins with exposing students to many contextual problems, current problems, and other hot problems. The problem exposed in the initial stage is usually word-problem along with direction to develop counter-argument. Counter-argument is defined as attribute of a good argumentation (Andriessen et al, 2003; Voss et al, 1991) and a standard for measuring an argument (Kuhn, 1991).

Problem based learning mostly provides students with opportunities to make argumentation. This learning is expected to guide students to reach the objectives of the learning namely the ability to make mathematical argument in a discussing a mathematical problems. This research took students of Mathematics education, especially those who take Calculus I. The reason why choosing this topic was that Calculus I provides many mathematical problems taken from real-life situation.

## **Method**

### **Participants**

Subjects in this research were students of mathematics education of UHAMKA University. They were chosen for their enrollment in Calculus I course – offered for freshmen. Other considerations when choosing these group of students were that they offered heterogeneity of academic skills, their level of reasoning, and their independent in learning. All these factors surely influenced the implementation of problem based learning.

The samples of this research were students at Mathematics education department of UHAMKA University who enrolled in Calculus I course. The samples were drawn using purposive random sampling for its excellence in providing randomized and more variant samples. All of the students were then divided into two groups: Experimental group and control group. In order to assure the randomized result of the samples, this research employed drawing techniques.

### **Research Procedures**

This research was a quasi-experimental study about the using of problem based learning. There were two group of student involved in this study in which their mathematical argument skill was measured in order to see if the problem based learning had successfully improve the students' mathematical argument skills. The first group called experimental group and they received treatment using problem based learning; and the second group called control group and they received treatment using conventional teaching method. In both groups, the students were then categorized into the following mathematical argument skills level as follows: higher, middle, and lower.

There were two stages in this research: identifying and developing learning components, and conducting the research that covered the whole planned process.

### **Instruments and Sources of Data**

This research employed several instrument to collect data. The instruments were as follows: (a) a test of students' initial skill level of mathematics; (b) a test of students' initial mathematical argument skill; (c) observation sheet; and (d) interview protocol intended to collect students' perspective towards problem based learning. Students' mathematical argument skill were measured through written test that consists of the following aspects: identification of assumption, identification of relevance and irrelevance data, analysis of argument, answer question with reasons (clarification), and give reasons on certain conclusion. Meanwhile, the test of mathematical argument skills consisted of eight items. These instruments were given to the students before and after the learning process.

### **Data Analysis**

This research analyzed the data using two approaches: quantitative and qualitative. Quantitative analysis consisted of descriptive and inferential statistics. The first step was descriptive statistics analysis included finding out the mean and standard deviation; then a diagram graphic was used to see the general description. In order to find out if there was improvement on students' mathematical arguments on both groups of samples, this research conducted an analysis on the results of pretest and posttest. Moreover, the data analysis was conducted by using the formula of average normalized gain (Hake, 2007).

The second step of data analysis was inferential statistics analysis that was carried out to test the hypotheses. This step was begun by conducting normality and homogeneity of variant both partly and wholly. Later, in order to find out if there were differences in the groups, to check if there was an interaction between free variable and controlled variable related to hypothesis, this research employed ANOVA with one line using SPSS-19.00 software with 95% level of trusted.

## Research Findings

This parts highlights the research findings. The analysis conducted in this research was intended to find out if there was interaction between the treatments, the students' initial level of skills of mathematical argumentation (PAM), and their mathematical argumentation skills.

**Table 1. Sample Distribution**

PAM Groups	Experimental Group	Control Group	Total
Upper	18	10	<b>28</b>
Middle	40	32	<b>72</b>
Lower	11	30	<b>41</b>
Total	<b>69</b>	<b>72</b>	<b>141</b>

Quantitative data was gathered from pretest intended to find out students' initial mathematical skills and their mathematical argumentation. There were 141 students taken the test in which 69 students in experimental group and 72 students in control group.

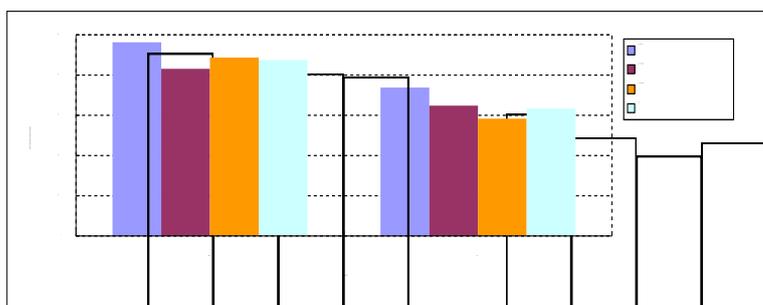
Picture 2 and 3 displays the scores of students' mathematical argumentation before and after the learning process. Meanwhile, the scores of students' mathematical argumentation for each PAM are summarized in Picture 2.



Notes: Ideal maximum score is 70

**Picture 2. Average of Students' Initial Mathematical Argumentation based on PAM Groups before Treatment.**

The following Picture 3 displays the scores of students' mathematical argumentation based on PAM Groups after the treatment.



Note: Ideal Maximum Score is 70

**Picture 3. Average of Students' Final Mathematical Argumentation based on PAM Group after the Learning Process**

The following Table 2 summarizes the improvement of scores of students' mathematical argumentation before and after the learning process.

**Table 2. The Description of n-Gain of Mathematical Argumentation Skills based on Learning and PAM Groups**

PAM	Learning Process									
	Problem based Learning					Conventional Learning				
	Minimum Scores	Maximum Scores	Mean	SD	n	Minimum Scores	Maximum Scores	Mean	SD	n
Upper	0,456	0,763	<b>0,618</b>	0,091	18	0,379	0,574	<b>0,455</b>	0,066	10
Middle	0,317	0,737	<b>0,521</b>	0,102	40	0,121	0,571	<b>0,377</b>	0,114	32
Lower	0,452	0,770	<b>0,575</b>	0,105	11	0,119	0,525	<b>0,320</b>	0,110	30
Total	0,317	0,770	<b>0,555</b>	0,107	69	0,119	0,574	<b>0,364</b>	0,115	72

The Table 2 above indicates that there is improvement in students' mathematical argumentation in which students in experimental group (Problem based learning) had better scores than those in control group (Conventional). The average scores (Mean) of n-gain of mathematical argumentation skill in experimental group is 0.555. it means that the mean scores of n-gain of mathematical argumentation of experimental group is higher than those in control group with only 0.36. In addition, the mean scores of improved mathematical argumentation in experimental group for all PAM groups (Upper, Middle, and Lower) is also higher than those in control group.

This general description indicates that there is a difference on mathematical argumentation skill between students in experimental and control group. Then, a statistical test was carried out to see if the difference is significant.

In order to see if there is difference on the mean scores of improved mathematical argumentation between experimental and control group, this research conducted *t*-test as displayed in the following Table 3.

**Table 3. *t*-Test on n-Gain Average Scores of Mathematical Argumentation Skill gained by Experimental and Control Groups**

Argumentative Skills	t-test			H0
	T	df	Sig.	Rejected
	10.174	139	0.000	

Before testing the hypotheses, all requirements must be met. The tested hypothesis was: H0: There is no significant difference on mean scores of students' mathematical argumentation skills gained by experimental and control group. Ha: The mean scores of students' mathematical argumentation skills gained by experimental group are higher than those in control group. The testing criteria said: if the probability scores (Sig.) is higher than 0.05, then the null hypothesis is accepted.

The following Table 3 shows that the null hypothesis is rejected. Therefore, it can be concluded that the improved mathematical argumentation skills gained by the students in experimental group is significantly higher than those in control group.

**Table 4. ANOVA Difference of Mean Scores (Average) of Improved Mathematical Argumentation based on PAM Groups and Problem base Learning.**

Difference resources	Sum of Squares	df	Mean Square	F	Sig.	H <sub>0</sub>
Between Groups	0.124	2	0.062	6.222	0.003	
Within Groups	0.656	66	0.010			Rejected
Total	0.779	68				

The n-gain data scores of students' mathematical argumentation both total scores and PAM based scores in experimental group are normally distributed and homogeneously varied. One line ANOVA was employed to find out if there was a difference in the improvement of students' mathematical argumentation based on PAM group and in problem based learning classroom. The summary of the ANOVA analysis can be seen in the Table 4.

Table 4 shows that probability scores (Sig.) is 0.003 which means that the average improvement of students' mathematical argumentation skills among the PAM groups in problem based learning classroom is significantly difference to one another. Then, this research conducted the Scheffe Test to find out which improvement that significantly difference in the students' mathematical argumentation skills. The summary of the test can be seen in the following Table 5.

**Table 5. Scheffe Test on the Average Scores of Improved Mathematical Argumentation based on PAM Groups in Problem Based Learning Classroom**

(I) PAM Groups	(J) PAM Groups	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval		H <sub>0</sub>
					Lower Bound	Upper Bound	
Upper	Middle	0.098*	0.028	0.004	0.027	0.169	Rejected
	Lower	0.043	0.038	0.526	-0.052	0.139	Accepted
Middle	Upper	-0.098*	0.028	0.004	-0.169	-0.027	Rejected
	Lower	-0.054	0.034	0.286	-0.139	0.031	Accepted
Lower	Upper	-0.043	0.038	0.526	-0.139	0.052	Accepted
	Middle	0.054	0.034	0.286	-0.031	0.139	Accepted

\*. The mean difference is significant at the 0.05 level.

The tested Hypotheses were as follows:

H<sub>0</sub>: There is no significant difference on mean scores of students' mathematical argumentation skills gained by experimental and control group..

H<sub>a</sub>: The mean scores of students' mathematical argumentation skills gained by experimental group are higher than those in control group.

The testing criteria said that if the probability scores (Sig.) is higher than  $\alpha = 0.05$ , then the hypothesis is accepted.

Based on the computation result in Table 5, it can be seen that there were probability scores (Sig.) for each pairs of PAM group: upper and middle, upper and lower, and middle and lower. The probability scores for the pair of Upper and Middle PAM group is 0.05 which means that the null hypotheses is rejected. Therefore, it can be said that there is significant improvement on mathematical argumentation skill of the Upper and Middle PAM Group. Meanwhile, the similar findings also happened in the pairs of Upper and Lower PAM group in which the probability scores is 0.05 and the null hypothesis is accepted. Therefore, it can be concluded that there is no significant different in students' mathematical argumentation skill between Upper, Middle, and Lower PAM groups.

This research also found that the overall data of students' mathematical argumentation skills were taken from each PAM Groups and gathered through normal distribution with homogeneous variants. This research then conducted a two lines ANOVA test in order to find out if there was interaction between learning process and PAM groups in mathematical argumentation skills.

According to the computation results of ANOVA as indicated in Table 6, it was found that the F Scores for PAM groups are 7.402 with probability values (Sig.) in 0.001. This figure means that PAM Groups have given significant effect on the improvement of students' mathematical argument skills and so is the factor of learning approach. It can be seen from the F scores of learning approach with 87.426 and probability value (Sig.) in 0.000 which means that this significant values are less than 0.05 thus making the null hypothesis is rejected.

According to the computation results displayed in Table 6, the F Scores for interaction between PAM Groups and learning approach is 3.190 with probability values (Sig.) in 0.444. This score is less than 0.05 thus the null hypothesis is rejected. Therefore, it can be concluded that the PAM Group factors and learning approach have jointly given significant impact on students' mathematical argumentation skills.

The following Table 6 summarizes the result of the two lines ANOVA test.

**Table 6. The Two Lines ANOVA between PAM Groups and Students' Mathematical Argumentation Skills**

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	H <sub>0</sub>
Corrected Model	1.545 <sup>a</sup>	5	.309	28.777	.000	
Intercept	24.501	1	24.501	2281.022	.000	
PAM	.159	2	.080	7.402	.001	Tolak
Learning	.939	1	.939	87.426	.000	Tolak
PAM * Learning	.069	2	.034	3.190	.044	Tolak
Error	1.450	135	.011			
Total	32.583	141				
Corrected Total	2.996	140				

a. R Squared = .516 (Adjusted R Squared = .498)

This research continued to conduct Scheffe test to find out which PAM group interacts with learning approach in improving students' mathematical argumentation skills. The test results are summarized in Table 7 below. The table shows that the improvement of students' mathematical argumentation skills in Upper PAM Group is better than in Middle and Lower PAM Groups. Meanwhile, the improvement of students' mathematical argumentation in Middle PAM group is better than in Lower PAM group. It indicates that problem based learning (PAM) has played significant roles in improving students' mathematical argumentation skill. In addition, the difference of to which degree the improvement occur between problem based learning classroom and conventional classroom is significant in which the probability values (Sig.) is less than 0.05. It means that there was interaction between learning factors (Problem based and conventional) and PAM Groups (Upper and Middle) in improving students' mathematical argumentation skills.

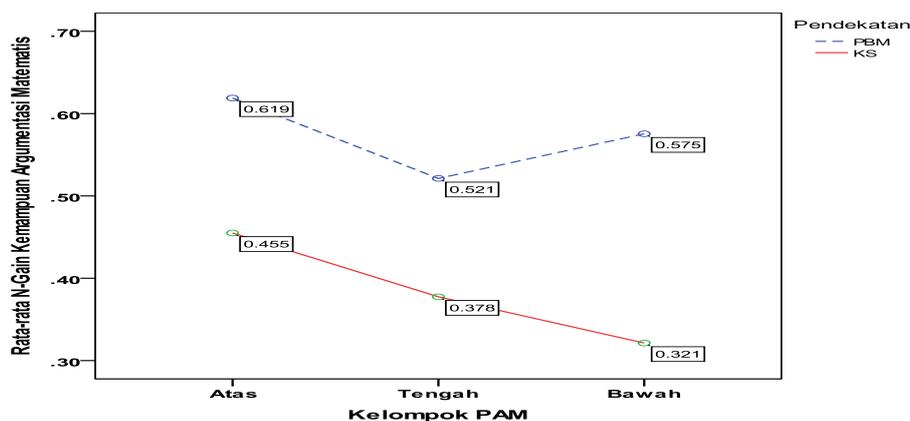
**Table 7. The Comparison of Improvement of Students' Mathematical Argumentation in both Problem based Classroom and Conventional Classroom**

(I) PAM Groups	(J) PAM Groups	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval		H <sub>0</sub>
					Lower Bound	Upper Bound	
Upper	Middle	0.103*	0.023	0.000	0.046	0.160	Rejected
	Lower	0.171*	0.025	0.000	0.108	0.234	Rejected
Middle	Upper	-0.103*	0.023	0.000	-0.160	-0.046	Rejected
	Lower	0.068*	0.020	0.005	0.018	0.118	Rejected
Lower	Upper	-0.171*	0.025	0.000	-0.234	-0.108	Rejected
	Middle	-0.068*	0.020	0.005	-0.118	-0.018	Rejected

\*. The mean difference is significant at the 0.05 level.

The research then conducted data analysis using graphic intended to find out the interaction between learning process and PAM Group in improving students' mathematical argumentation skills. The graphic had helped this research in identifying the interaction as it can be seen in Picture 4.

Picture 4 above illustrates the interaction between learning factors (Problem based Learning and Conventional Learning) and the PAM groups in improving students' mathematical argumentation skills. This interaction happens due to the gap in the improvement of students' mathematical argumentation between problem based classroom and conventional classroom. Therefore, it can be concluded that there were interactions between learning factors (Problem based learning and conventional learning) and PAM Groups (Upper, Middle, and Lower) in improving students' mathematical argumentation.



**Picture 4. The Interaction between Learning Approach and PAM Groups in Improving Students' Mathematical Argumentation Skills**

## Discussions

This research has successfully found that there was significant improvement in students' mathematical argument skills in which students in experimental group have better improvement than those in control group. Table 2 highlights the average of improvement of the argumentative skills where students in experimental group gained the scores of 0.555 (moderate improvement) and students in control groups gained the scores of 0.364 (Light improvement). This improvement is supported by the statistical test results which indicate that mathematical argumentation skills of students in problem based learning classroom is better than those in conventional learning classroom.

The improvement of students' mathematical argumentation skills happens due to the implementation of problem based learning that exposes students with argumentative skills. The more often the students practice their argumentation skills, the more skillful they in making argumentations. Osborne (2005) said that making an argument as a long process that needs experience and significant practice repetitively. In addition, the improvement of argumentation skills also happens due to the opportunities that problem based learning provide to the students have to understand basic knowledge, facts, and application and the ability to communicate effectively and accurately both oral and written, and to work together in small and big groups (Duch et al 2001).

In average, the improvement scores of the students' mathematical argumentation skills are 0.618 for Upper PAM group, 0.521 for Middle PAM Group, and 0.575 for Lower PAM group to which all of them considered to be moderate improvement category. The improvement of the mathematical argumentation skills gained by students in problem based learning class for Upper PM group is better than those in Middle and Lower PAM group. But, students in Middle PAM group did not get better improvement than those in Lower PAM group. In average, students in Middle PAM group get lowest improvement among all PAM groups.

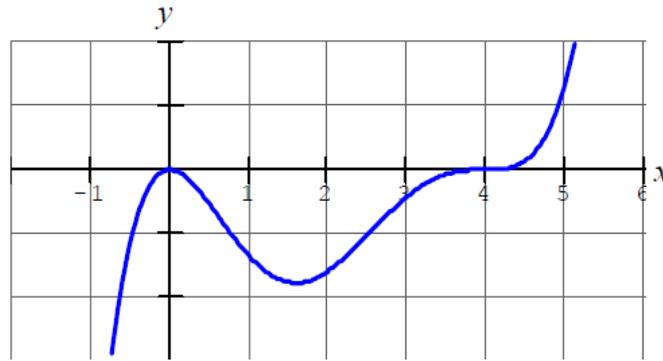
Meanwhile, in the conventional classroom, the average improvement of mathematical argumentation skills for Upper PAM was 0.455, for Middle PAM was 0.377, and 0.320 for Lower PAM group, and this improvement can be categorized into light improvement. In addition, students in Upper PAM get the highest improvement among three PAM groups.

In the problem based classroom, there was significant difference of improvement gained by student in the three groups (Upper, Middle, and Lower). The most significant difference occur between Upper and Middle group. This research also found that improvement gained by students in Middle group of Problem based classroom is higher than the upper group in conventional classroom. This is based on the data analysis results which indicated that the improvement of Middle group in Problem based classroom was 0.521; meanwhile in the Upper group of conventional classroom, the scores were 0.455. And the improvement in Middle PAM of problem based classroom and Upper PAM in conventional classroom can be categorized into light/moderate improvement.

Based on the argumentation level, the Toulmin model of students' responses on the question Number 7a is in the level of 5.

**Question Number 7a.**

Look at the following  $f(x)$  function:

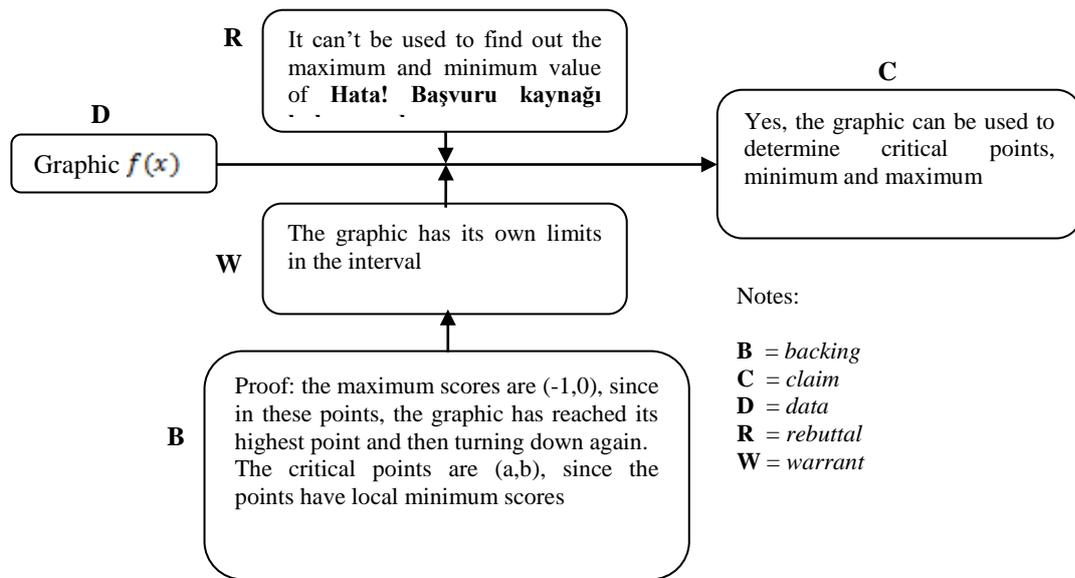


a) If the graphic displayed is graphic of  $f(x)$ . Can the graphic be used to determine critical point, local maximum and minimum, and absolute minimum of  $f(x)$ ? Provide your reasons!

The students are able to understand the concept of minimum and maximum of a curve. Students' responses indicate that they have already possessed the pathway of thinking since it consisted of data, claim, warrant, backing, and rebuttal.

- Data : *Graphic based questions*
- Claim : *Yes, the graphic can be used to determine critical point, maximum, and minimum.*
- Rebuttal : *But it can't be used to find out the absolute value of minimum and maximum of  $f(x)$ .*
- Warrant : *The graphic has its own limit in each interval..*
- Backing : *Proof: the maximum score is (-1,0), since in the point, the graphic reach the highest point before turnig down again. The critical points are (a,b), since in these points, the graphic has its local minimum scores.*

The following picture visualizes the students' responses:



The results of data analysis have informed that there was interaction between PAM groups (Upper, Middle, and Lower) and learning factors in improving students' mathematical argument skills. Both PAM groups and learning factors have given impact significantly on the improvement of mathematical argumentation of the students. The findings of the research indicate that there is a huge possibility to implement problem based learning in improving students' mathematical argumentation skills.

In addition, this research also found that there was difference improvement of argumentation skills between students in the three PAM groups. The most significant different is between Upper and Lower PAM group. It indicates that problem based learning is appropriate to implement in Upper PAM group. As it can be seen from its characteristics, through learning in a problem based classroom, students in Upper PAM group can optimize their communication skills to be more effective and accurate both verbal and non-verbal. Students who have higher initial mathematical skills (Upper group) have had experienced activities intended to solve mathematical problems and in real-life situation. Thus, they are able to make a claim through logical analysis and be able to provide supporting logical evidences.

## Conclusion

In conclusion, the improvement of students' mathematical argumentation skills in problem based learning classroom is better than those in conventional learning classroom. The average improvement gained by students in all groups (Upper, Middle, and Lower) in problem based learning are significantly different to one another. Moreover, by using Scheffe Test, this research identified that there was significant difference in the improvement between Upper group and Middle group. The factors of PAM group have significantly given impact on the improvement of students' mathematical argumentation skills. There are also interactions between learning factors and PAM group. Looking at the scores, problem based learning found to be more appropriate to students in Upper and Lower group in improving students' mathematical argumentation skills.

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