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Information on Coordinate System as a Tool for Developing Mathematical Thinking

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Abstract

The present paper presents activities tested and tried in class, which are designed to promote and develop mathematics knowledge, based on visual illustrations. It presents activities designed to stimulate creative thinking while developing mathematical thinking and more thorough comprehension of the meaning of information in an axes system. Activities like those, as a multi-disciplinary practice, will entail pupils' better and more meaningful learning of mathematics and will enrich knowledge of this subject. Furthermore, being well acquainted with unusual graphic representations will greatly enhance the study of algebra and other mathematics subjects later on.

Key words: Coordinate System, Graphic representation, Meaningful learning, Creative thinking

Introduction

Everyday events or processes can be presented in various ways; one of them is graphic representation by axes. There are different graphic representations which enable us to get a visual picture of the information as well as an option of using them for making calculations, comparison of sizes and shifts from one representation to another.

Information presented by an axis offers numerous options of mathematical activity at different thinking levels, starting at elementary school where pupils are first exposed to the axes and up to the high grades of high school. Moreover, introducing technology into mathematics study enables expansion of computer-aided mathematical activities, thus facilitating graphic drawings in an axes system.

In the curriculum and in textbooks, axes are mainly presented with numerical values. In this way learners are exposed to the meaning of the points indicated on the graph. Reading information from such a system constitutes an initial basis for comprehending the values of the points and the graphic descriptions.

This paper presents activities designed to stimulate creative thinking while developing mathematical thinking and more thorough comprehension of the meaning of information in an axes system. This is done without any numerical values on the axes as is prevalent in most textbooks. Furthermore, being well acquainted with unusual graphic representations will greatly enhance the study of algebra and other mathematics subjects later on.

Theoretical Background

Developing mathematical and creative thinking within the framework of mathematics lessons

Research of mathematics education (Gazit & Patkin, 2009; Leikin et al., 2009; Shriki, 2010; Levav-Waynberg, & Leikin, 2012) reinforces the need for seeking various ways of developing mathematical thinking and creativity. One of the ways of promoting this issue is the presentation of exceptional activities which require 'out of the box' mathematical thinking.

Teaching mathematics at elementary school usually focuses on the inculcation of algorithms and through them acquaintance with logical thinking which complies with the rules of mathematics. All these leave mathematics

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teachers a little time and latitude for open-ended activities which facilitate expression of creative thinking. In fact, only when teachers' objective is to develop higher-order thinking should they implement even some of the recommendations made by Resnick (1987). For example: design assignments which allow learners to find several solutions, activities requiring use of different criteria or activities which are not algorithm-based.

In his paper, Mann (2006) reviews the different definitions of creativity in mathematics. He argues that when learners graduate, they are adequately capable of calculating and performing algorithms but are unable to apply this knowledge in a meaningful way. According to him, teaching mathematics with no investment in the promotion of creativity does not allow learners to see the beauty of mathematics. Moreover, it prevents talented and gifted learners from developing their special skills in this subject.

To sum up, the frameworks of mathematical activities prevalent in class and their levels are dictated by the textbooks. The way they are presented to learners depends mainly on mathematics teachers. The higher teachers' awareness of developing mathematical and creative thinking is, the better they will know how to cope with the challenge.

Graphic representations and the teaching

Since the beginning of the 21st century, graphic representations have become much more common and available in any field, not only in mathematics but also in other sciences. New technological tools allow an almost immediate translation of information about varied subjects into different graphic representations. Shah & Freedman (2011) showed that between the years 1984-1994 the number of graphic representations in academic magazines has doubled. This is also demonstrated in daily newspapers. Graphs have turned into the tangible, available and easiest 'tool' for presenting information of all types. Consequently, it has become necessary to enhance the issue of teaching graphs at schools and higher education institutions as part of the inculcation of scientific literacy.

Moreover, scientific research of graphs has increasingly expanded. For example, Berg & Smith (1994) investigated the way learners read and explain information presented by a graph. They identified a significant relation between learners' logical thinking capability and graphing skills. Other studies (Canham & Hegarty, 2010; Cook, Wiebe & Carter, 2008; Hipkins, 2011) explored what affects learners' mastery in graphs. The findings illustrate that in addition to the learners' thinking capability, educators' teaching strategies and learning the use of computers while teaching the subject are also important. All these factors combined together affect learners' capability of creating new graphs and explaining the information presented to them.

The study conducted by Carpenter & Shah (1998) presents the three stages of becoming versed in the issue of graphs. First, learners should be acquainted with the visual part, namely the representative shape of the graph, the importance of the shape and its representative features. Second, they have to know the numerical and quantitative components and what they represent. Third, they should connect the quantitative and graphic part of the conclusions.

As to the determination of learners' knowledge of graphs, Wainer (1992) suggests the type of questions which can be asked: Basic level questions which relate to data extraction; intermediate level questions which engage in trends emerging from the information; and high level questions which deal with the depth of the information meaning while comparing trends.

To sum up, the research literature (Friel, 2001; Shah & Freedman, 2011; Wang, 2012) classifies the information presented by graphs into three types which also represent the various thinking levels required by learners.

1. Explicit information – this information is usually presented already in the axis names and it is easily identified.
2. Tacit information – for obtaining this information, learners should examine more thoroughly what is represented by the graph. For that purpose mathematics knowledge (basic or advanced) is necessary.
3. Conclusive information – the highest level of information which requires reasoning and justification by means of mathematical tools in order to draw the conclusions.

Acquaintance with René Descartes as an introduction to the teaching of the Cartesian Coordinates

“*Cogito ergo sum*” – I think, therefore I am (René Descartes)

René Descartes, the French mathematician and philosopher (1596-1650), was the first to come up with the idea of accurately presenting the location of points on the plane and later in space. He conceived the idea while serving in the army as an officer on the banks of the Danube. Some say that the innovation was created following a dream-vision. Descartes delayed publishing his invention and only in 1637 he printed his treatise *Discourse on the method* – a method through which he presented his philosophical concept “The method of leading intelligence in the right way”. At the end of the treatise there are three appendices. The third appendix, which is 106-page long, is entitled: “Geometry” and includes Descartes’ greatest contribution to science: every point on a plane can be presented by means of a pair of real numbers. The mathematician and philosopher Leibnitz invented calculus at the same time as Newton during the second half of the 17th century and called it on the name of the person who conceived it: ‘Cartesian Coordinates System’ (Gazit, 2004).

The “Cartesian Coordinates System” – is derived from Descartes, a name which is pronounced ‘Cartesius’ in Latin. The Cartesian Coordinates System is defined by two straight lines called axes which are usually located at a right angle. The horizontal axis is indicated by X and the vertical axis by Y. The point of intersection of the axes is called *origin of the axes*. In order to indicate a certain point on the axes system, we indicate the values of the X and the values of the Y on the point, thus creating the ordered pair of numbers(X,Y). The expression “Cartesian Coordinates System” serves only in 2 and a 3-dimensional space.

This system paved the way to analytical geometry and modern mathematics. It is inconceivable to imagine the existence of numerous developments in mathematics and their implementation without the Cartesian Coordinates System. Hence, we should start teaching this subject by first presenting Descartes and his extensive contribution to mathematics in general and to the subject of axes in particular. The narrative behind the invention of the Cartesian Coordinates System and their mathematical development in future generations should be the basis for the study of this essential subject. Teachers can tell the learners about Descartes who saw a fly on the wall and conceived the idea of an axes system as a mathematical tool by means of which one can present the walking course of the fly and the possibility of presenting at any given moment its location on the wall.

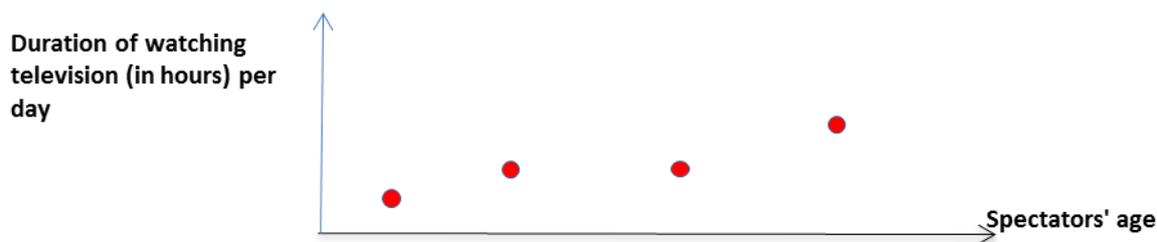
Examples of activities based on reading information from a graph

The following activities promote and enhance mastery in reading information presented by axes. The activities bridge between mathematics learnt in class and occurrences in the learners’ environment. Moreover, this kind of activities can facilitate better understanding of the topic of functions which will be studied later.

For the purpose of presenting information or reading it from a graph, learners need first to be acquainted with axes and understand the meaning of the graph and its various representations. In order to draw conclusions based on a graphic representation, mathematics knowledge as well as logical thinking capability are also required.

Preliminary activity:

Please write in words what is presented in the graph below, your conclusions and insights.



Acquaintance with and comprehension of the above graph while writing or verbalising things enables moving to the next activities which are more complex and require coping with higher-level assignments.

Suggestions for teaching the subject in class:

- Ask the learners to ‘tell’, describe in writing or orally what the narrative of the graph is.

- b. Ask the learners to 'tell', describe in writing or orally each of the assertions and indicate which of them are correct and according to which graphs.
- c. Ask the learners to raise additional questions for a given graph.
- d. Present examples from the press or from everyday life in order to describe a real graph.

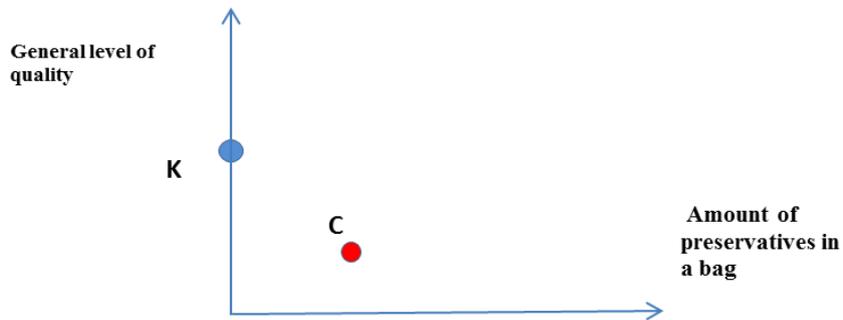
Activity No. 1 is the easiest and requires only initial and basic knowledge of reading information. Activity No. 2 requires a higher-level thinking as well as a capability of formulating assertions and drawing conclusions about a graph which is not explicit. Here too the issue of creative thinking development is integrated.

Activity No. 1

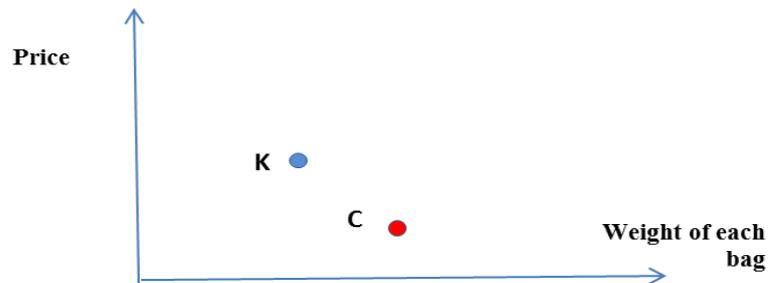
Below are graphs which describe information about two snacks, 'Aces' and 'Champions'.

The 'Knight' snack bag is marked with the letter **K** and the 'Champions' snack bag is marked with the letter **C**.

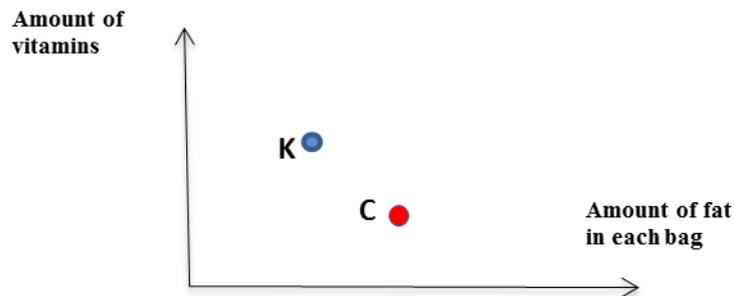
Graph No. 1:



Graph No. 2:

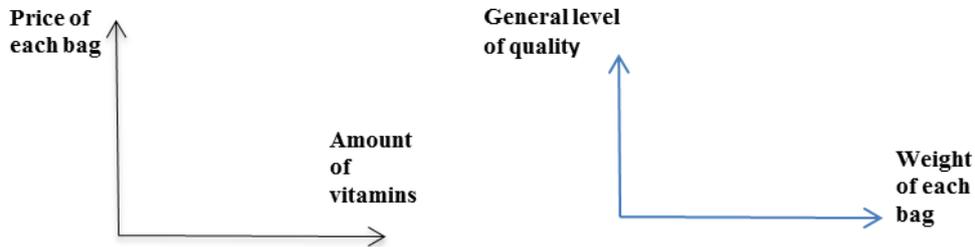


Graph No. 3:



- I. Check each of the following assertions and indicate which of them are correct and according to which graphs.
 1. The 'Knight' snack is cheaper than the 'Champions' snack.
 2. The 'Knight' snack bag weighs more than the 'Champions' snack bag.

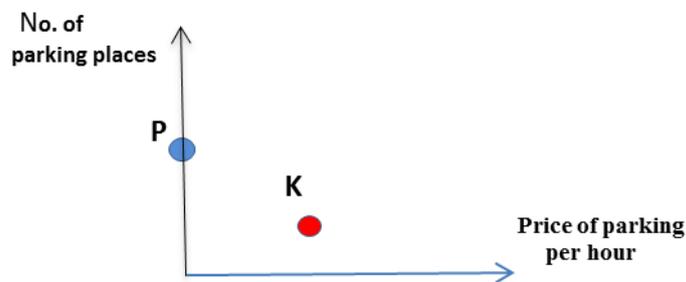
3. The `Knight` snacks are better for weight watchers.
 4. The `Knight` snack has more vitamins than the `Champions` snack.
 5. The `Knight` snack is of higher quality than the `Champions` snack.
- II. Please indicate the location of snacks K and C in the axes system, according to the information obtained from the previous graphs.
- III.



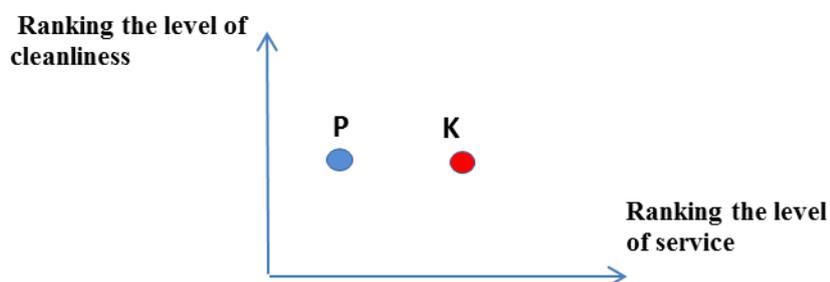
Activity No. 2.

The following graphs describe facts associated with two shopping centers, `Knights` Mall and `Princes` Mall. The **Knights` Mall** is marked with the letter **K** and the **Princes` Mall** is marked with the letter **P**.

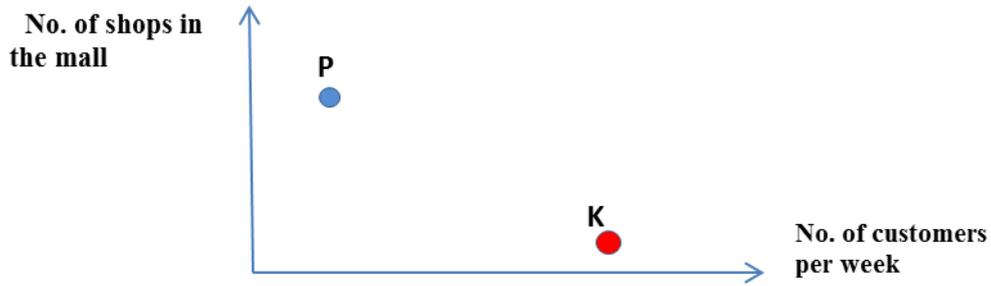
Graph No. 1



Graph No. 2



Graph No. 3



- I. Check each of the following assertions and indicate which of them are correct and according to which graphs.
 1. In the 'Princes' Mall the level of cleanliness is lower than in the 'Knights' Mall.
 2. In the more popular mall there are more parking places.
 3. In the mall where the level of service is low, parking is free of charge.
 4. In the mall with a small number of parking places, the number of customers is higher.
- II. Here is another fact: in the mall where the number of shops is greater, the prices are higher.
- III. Use the information in the previous item and in the previous graphs in order to indicate the malls (points P and K) in the following axis systems.



- a. Based on all the information you have about the two malls, please formulate three assertions which are correct and three assertions which are incorrect.
- b. Which assertions were easier to formulate, the correct or the incorrect ones? Please give reasons for your answer.

Discussion and Conclusions

The activities described above were given to learners in elementary school (5th – 6th grades -42 pupils), junior high school (7th grade- 24 pupils) and pre-service teachers (35 students at the first year of study).

- a. All three group of population got the same activities.
- b. Despite the fact that the pre- service teachers had much more mathematics knowledge then the school pupils, for most of them this was a new experience.
- c. They were versed in reading 'non-numerical' information yet they found it difficult to decide whether the assertions were correct or not.
- d. All three groups' encountered difficulties in creating new graphs based on prior information.
- e. The next assignment for all the research populations was creating a similar activity about a different subject.
- f. Most of the school learners re-wrote the presented activities with slight changes while only 7 pre-service students designed activities which were harder and more complicated than the ones presented to them.

To sum up , learning the subject not by calculating numerical values stimulated development of logical thinking by formulating assertions and practicing 'graphic literacy', which they had not experienced before. The shift from a preliminary and easy activity of reading information on a graph to more complex activities following learners' being asked to design activities by themselves in this spirit actually integrate the two aspects – developing mathematical thinking and creativity.

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