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A Discussion about Errors in Algebra for Creation of Learning Object

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Abstract

This Article reports the results of a research with 333 freshmen students of Differential Calculus, for whom it was applied a test with questions about basic mathematics. The question analysed in this article involves basic algebra. The students made mistakes in operations and algebraic properties that are essential for the continuity of their studies in Calculus, especially to solve exercises of limits and derivatives. Then, we sought to some theoretical constructs to discuss the errors, such as concept image, symbol sense, structure sense and algebraic insight. The main difficulties observed are related to the distributive property of multiplication over addiction. In this paper we propose the creation of a learning object, in accordance to the principles of multimodality to help students overcome their difficulties in algebra.

Key words: Error analysis, Distributive property, Algebra learning, Elementary school

Introduction

Freshman students in higher education courses in the field of Exact Sciences have shown several difficulties in Differential and Integral Calculus. Studies carried out in several institutions of Higher Education, in Brazil and in other countries, indicate that the contents studied in the previous years are the main source of errors in resolution of exercises and problems (Tall, 1992; Porter & Masingila, 2000; Artigue, 2004; Cabral & Baldino, 2004; Hardy, 2009). The researchers reported the problems to work with Differential Calculus presented by the students, who study in a private university in Rio Grande do Sul state, Brazil. An investigation was developed to analyze the learning difficulties the students presented in the Differential Calculus classes and the possibilities to overcome those difficulties through technological sources.

In the first classes of Calculus, first term of 2013, a diagnostic test was administered to evaluate the students' knowledge that is needed in the limits and derivatives studies. The results showed that difficulties have origin in the algebra learnt in elementary school, mainly in the use of distributive property of multiplication over addiction. The analysis of the mistakes made by the students in the first question of the test are presented and discussed in this article. A learning object is suggested to help the students to overcome their difficulties.

Since the analysis of the responses revealed the prevalence of errors related to the distributive property, we sought some constructs, developed by researchers of algebra teaching, to discuss the results obtained. Among these constructs, we highlight transparent and opaque representation (Zazkis, 2005), visual salience (Kirshner & Awtry, 2004), symbol sense (Arcavi, 1994), structure sense (Hoch & Dreyfus, 2004) and algebraic insight (Pierce & Stacey, 2001, 2004), later presented.

Theoretical Contributions to the Discussion

The main probable cause of errors in the correct application of distributive property of multiplication over addiction is due to how the student was taught this property. When the teachers want to introduce algebra to the students in elementary school, they usually present the definition and the schematic model shown in Figure 1.

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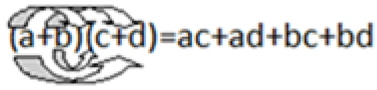

Figure 1. Schematic of distributive property

Thus, each student is able to form an individual concept image, including a scheme as the Figure 1, some definitional elements (more or less memorized, according to the time devoted to study) and some examples. It is important to explore examples that include difficulties for the application of distributive property, so that in another moment of the learning process the student do not remember only notions of concept image, which become potential factors of cognitive conflict. The notion of concept image is used as defined by Tall and Vinner (1981, p . 152): "[...] The total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing the individual meets new stimuli and matures".

The pedagogical approach to present a concept, as the distributive property, should include a large variety of representations. According to Tall (2000, as cited in Giraldo, 2004), each representation draws attention to a specific characteristic of the concept and obscures others. Zazkis (2005) distinguishes between transparent and opaque representation. In the representation of distributive property, as shown in Figure 1, there are transparency in the way the terms should be operated, but the representation is opaque about the signal of the terms.

Kirshner and Awtry (2004) presented the constructor named visual salience; it was characterized by them as “aesthetic sense of form”. According to them, students tend to make incorrect transformation patterns of the expressions, as the following: $(a+b)^c = a^c + b^c$; $\sqrt[c]{a+b} = \sqrt[c]{a} + \sqrt[c]{b}$; $a^{m+n} = a^m + a^n$; $\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$. Authors consider that these errors seem to have a superficial character and, instead of showing a misunderstanding of correct rules, they demonstrate an incorrect perception of those correct rules. In fact, according to the authors the incorrect rules are due to a overgeneralization of the correct rules explained in regular mathematics classes.

The visual salience rules have a coherence that makes both sides of the equation seem to be related. Besides, Kishner and Awtry (2004) consider that the repetition of the elements and their visual separation in both sides of the equation also contribute to the visual salience. For example, once students know the distributive property scheme they might overgeneralize it to any other equality which its elements seem to be distributed related to the others. That is the case of the pseudodistributive of the first wrong rule, mentioned earlier in this paper, which is a overgeneralization of (a+b)c=ac+bc.

Symbol Sense is another expression mentioned in several articles about algebra. Pierce and Stacey (2004) consider that there were two relevant attempts to describe the symbol sense. Those were made by Fey (1990) and by Arcavi (1994). Arcavi goes beyond Fey describing and discussing examples that illustrate the symbol sense. Some considerations and examples are shown later in this study.

The student needs to be familiar with the symbols, which requires an understanding of what and how the symbols have to be used to solve the problems, and also, when to choose other solutions, not algebraic. It was also mentioned by Arcavi (2004) the importance of not use algebraic manipulation when other representations show simpler solutions. This case is the problem: *what values can x take in* $|x-2|>|x-6|$? An algebraic solution which involves inequalities demands a technical work and it has a high probability of mistakes to occur. In this case a draft of the graph of the functions given by $f(x)=|x-2|$ and $g(x)=|x-6|$ allows an immediate solution.

Moreover, Arcavi (1994) said that knowing the algebraic manipulations to solve problems it is not enough, in some cases it is necessary to understand the meaning of the symbols. The author also identifies four key behaviors: reading instead of manipulation of the symbols; reading and manipulation; reading as the goal for manipulation, reading for reasonableness.

The first behavior can be illustrated with the data from an investigation made by Hoch and Dreyfus (2004), in which high school students were asked to solve the equation $(1-\frac{1}{n+1})-(1-\frac{1}{n+1})=\frac{1}{132}$. They found the common denominator of the terms of the first member of the equality and made use of algebraic manipulation, instead of notice that the first member of the equation is zero, so there is no solution. Thus, interrupt the manipulation, read and understand the symbolic relationship in this case is part of the symbol sense.

To the second behavior, Arcavi (2004) presents an example: if a student has the symbol sense, he or she can notice that the equation $\frac{2x+3}{4x+6}=2$ has no solution, because the algebraic fraction numerator of the first member is half the denominator. If the student insists on manipulate the equation, he or she will get to $= -\frac{3}{2}$, which is exactly the value that may not be used to replace x in the equation.

The third behavior can be illustrated with an exercise of demonstration by induction. We asked to algebra students in a mathematics teachers training course to solve the following problem: "The proposition $n!>n^2$ is not valid to any natural number greater than zero. From what value of $n \in N$ it becomes real? Express the real relationship and prove it by induction." The resolution requires more than just the standard procedure of induction method, it is necessary to read to understand what is being asked, and then follow the procedures of induction to show that the inequality holds for $n \geq 4$.

The 4th behavior, according Arcavi (1994), is to develop the habit of rereading and testing (for replacements, for example) a particular expression obtained when trying to solve a problem rationally. In a test of the Integrated Algebra of Regents High School Examination 2010, there is the following problem: "Find three consecutive positive even integers such that the product of the second and third integers is twenty more than ten times the first integer ". (p. 22). If the student is in doubt about the algebraic expression that represents the situation, he or she can test values to check if the answer is correct, since the passage from natural language into mathematical language is a source of error in this type of problem. The algebraic manipulation is a skill that we aim to develop when we teach algebra. The cases reported by Arcavi (1994) help us to reflect on the proposed mechanic resolution from a list of standard exercises, which do not always develop the symbol sense.

Hoch and Dreyfus (2004) bring the concept of " structure sense ". They start asking if two algebraic expressions that have the same structure are equivalent and give, as an example, 30x 2 -28x+6 and (5x-3) (6x-2). As we can see, the first is a quadratic expression and the second is a product of two linear factors. However, they both have the same structure and find out which of them use in a specific context is part of what these authors define as structure sense. That can be described as a set of skills that include, among other aspects, the vision of an algebraic expression or sentence as a single entity, the recognition of an algebraic expression as a structure previously studied and manipulations that can be used on it.

Another construct that has been discussed among researchers of algebra teaching is the one Pierce and Stacey (2001) call "algebraic insight". The authors refer to the work as Computer Algebra System (CAS) and consider that the algebraic insight is a subset of symbol sense that can be defined as "the algebraic knowledge and understanding which allows a student to correctly enter expressions into the CAS, efficiently scan the working and results for possible errors, and interpret the output as conventional mathematics". (Stacey, 2001, p. 418-419).

According to Pierce and Stacey (2004), the algebraic insight, has two aspects: "algebraic expectation" and ability to link representations. The term algebraic expectation is used "to name the thinking process that takes place when an experienced mathematician considers the nature of the result they expect to obtain as the outcome of some algebraic (and symbolic) process". (p. 5). For example, when it is decided that two expressions are possibly equivalent without doing any calculation or algebraic manipulation.

In a framework with elements and common instances of algebraic insight , Pierce and Stacey (2004), divide the algebraic expectation into three elements: a) recognition of convenctions and basic properties, which common instances are the knowledge of the meaning of the symbols, the order and the properties of operations; (b) identification of structure, which common instances are the identification of objects and of strategic groups of components and recognition of simple factors ; c) identification of key features, related to the identification of the form and the dominant term, as well as the union of the form with the type of solution.

Pierce and Stacey (2004) commented that fundamental mathematical difficulties may be better detected by the teacher when the student works in a CAS. The authors mention the example of a student who worked with the function given by $f(x) = \frac{x^2+1}{2-x}$ and that typed x^ 2 +1/2-x, obtaining, on the computer screen, the image of the function x^2 + ½ - x, which surprised her. Later, working with the expression $\frac{x^2-1}{x-1}$, the student, after finding her mistake, entered correctly the parentheses, by writing (x^ 2-1) / (x-1). However, she was once again surprised, when obtained, as result shown by the machine, the binomial x+1.

Arcavi (1994) remade the list proposed by Fey (1990) and considers that the symbol sense must include, among others, an understanding of and an aesthetic feel for the power of symbols, which brings the idea of visual salience, by Kirshner and Awtry (2004); an ability to manipulate and to "read" symbolic expressions as two complimentary aspects of solving algebraic problems. This meets aspects presented by Hoch and Dreyfus (2004) when they define structure sense: the awareness that one can successfully engineer symbolic relationships which express the verbal or graphical information needed to make progress in a problem, which recalls the definition of algebraic insight (Pierce & Stacey, 2001). Thus, we can say that these different constructs presented by algebra teaching researchers form the framework for the work with the difficulties of the students.

Methodology

In this investigation it was used qualitative methodology. A probing test was planned, composed of five open questions regarding the content of mathematics in elementary education. The test was applied to 333 freshmen students of Differential Calculus, by the teachers responsible for the classes, on the first day of class. After the application of the test, the answers were the object of analysis of the content of the errors, performed based of the proposal of Bardin (1979), in three phases: pre-analysis, material exploration and treatment of results.

In the first phase the question chosen was: *Solve, in* ℝ, *the equation:* $\frac{2(x+3)}{3} - \frac{x-4}{2} = 2 - \frac{x+1}{4}$. Each response was indicated by a letter and a number, the initial letter of the teacher's name and the student´s number. The answers were corrected and classified into correct, partially correct, incorrect and blank.

In this first phase of the analysis, the 333 answers to the question chosen formed the *corpus* to investigate the types of errors (Bardin, 1979). In the second phase, the exploration of the material, the incorrect answers were unitarized and classified. In the third phase, the treatment of the results, the categories were described and illustrated.

Results

Among the 333 answers, 46 (14%) were correct, 9 (3%) parcially correct and 252 (76%) were incorrect. To analyse the 252 incorrect answers, we chose to classify only the first error committed by the student. Many times there was, for example, an initial error in the reduction to the same denominator and then the same student has committed an error of calculation, but the type of error classified is related to the reduction to the same denominator.

The categorisation of incorrect responses was made in two stages; initially 16 classes were created, and after the review of the types of errors, those were refined, obtaining at the end, five classes, herein after described and exemplified. The answer was typed as it was presented, so that, with the initial error and the expanding, that may be correct or present more errors.

Class I: the answers which the student shows difficulty regarding to the distributive property of multiplication over addiction are part of this category of answers. As an example, we have the answers of students T20 and G49, indicated, respectively, in Figures 2 and 3, below. All the examples were typed, because the answers were written in pencil and were not legible on scanning.

$$\frac{2(x+3)}{3} - \frac{x-4}{2} = 2 - \frac{x+1}{4}$$

$$2x + 2 - x - 2 = 2 - \frac{x+1}{4}$$

Figure 2. Student T20 answer

$$\frac{2(x+3)}{3} - \frac{x-4}{2} = 2 - \frac{x+1}{4}$$

$$\frac{4x+12-3x-12}{6} = 2 - \frac{x+1}{4}$$

$$\frac{x}{6} = 2 - \frac{x+1}{4}$$

$$\frac{2x+3x+3}{12} = 2$$

$$\frac{5x+3}{12} = 2$$

$$5x + 3 = 24$$

$$5x = 27$$

$$x = \frac{27}{5}$$

Figure 3. Student G49 answer

Class II : are part of this category the responses in which the student shows not knowing reduce fractions to the same denominator , such as, for example, the student G33 answer, indicated in Figure 4:

$$\frac{2(x+3)}{3} - \frac{x-4}{2} = 2 - \frac{x+1}{4}$$

$$\frac{2x+6}{3} - \frac{x-4}{2} = \frac{-x+1}{8}$$

$$\frac{4x+12-3x-12}{6} = \frac{-x+1}{8}$$

$$\frac{4x-3x}{6} = \frac{-x+1}{8}$$

$$\frac{16x-12x=-24x+24}{24}$$

$$4x + 24x = 24$$

$$28x = 24$$

$$x = \frac{24}{28} \quad \frac{12}{14} \quad \frac{6}{7}$$

Figure 4. Student G33 answer

Class III: the answers that present calculation errors are understood as errors in operations in elementary operations. An example is the C27 student's answer, in Figure 5, which was wrong at the beginning, the product 4.2.3, indicating as 48.

$$\frac{2(x+3)}{3}-\frac{x-4}{2}=2-\frac{x+1}{4}$$

$$\frac{8x+48}{12}-\frac{6x-24}{12}=\frac{24}{12}-\frac{3x+3}{12}$$

$$8x+48-(6x-24)=24-(3x+3)$$

$$2x+48+24=24+3x-3$$

$$3x-2x=48+3$$

$$x=51$$

Figure 5. Student C27 response

In this case, the error is arithmetic and shows difficulties with the number sense.

Class IV: the incomplete responses are in this class, in which the student solves only one of the two members or both, but is not able to complete. Also the answers in which the student, after solving correctly both members, moved all the terms for the first member and did not equal it to zero, leaving only an expression and not an equation. For example, the student M40 presented the response indicated in Figure 6:

$$\frac{2(x+3)}{3}-\frac{x-4}{2}=2-\frac{x+1}{4}$$

$$\frac{4x+12-3x+12}{6}=\frac{8-x-1}{4}$$

Figure 6. Student M40 answer

Class V: here are the answers that present errors committed by only one student, so they do not constitute a separate category, but present some detail that deserves to be analysed. The example below is the student I2 answer, indicated in Figure 7:

$$\frac{2(x+3)}{3}-\frac{x-4}{2}=2-\frac{x+1}{4}$$

$$4\left(\frac{(2x+6)}{3}-\frac{x-4}{2}\right)=2-x-1$$

Figure 7. Student I2 answer

According to this classification, we found that 60% of the incorrect answers were in Class I, 23% in Class II, 9% in Class III, 6% in Class IV and 3% in Class V.

Discussion

Analyzing the answers exemplified, we observe that the student T20 "canceled" the fractions on the left side of equality, the number 3 of the numerator with the 3 of the denominator in the first fraction, respectively, and the

number 4 of the numerator with the 2 of the denominator in the second fraction. It means that the student did not understand that to "cancel" a term in a sum or difference, it is necessary a common factor that can be placed in evidence. Possibly the unexpected result of this first error led the student to leave the question, and only reducing (incorrectly) the similar terms on the left side.
The right side of the equation as indicated in Figure 1, which represents the law of distributive property, is not often used in class, and sometimes this property is not even identified. More specifically, the equality is rarely read from right to left, so that " factor out the common terms" is not seen as distributive property but as a case of factorising. On the left side of the equation $\frac{2(x+3)}{3} - \frac{x-4}{2} = 2 - \frac{x+1}{4}$, the student T20 did not recognize the lack of the number 3, in the first fraction, nor the lack of number 2, in the second fraction, as common factors to make the "cancellation" possible with their respective denominators. Thus, T20 shows not having algebraic insight, because does not recognize conventions and basic properties that allow the "cancellation" of terms.

The student G49 committed the first error reducing the fractions on the left side of the equality to the same denominator 6, therefore, to multiply the second fraction by 3, the student did not realise that the minus sign in front of this fraction indicated (-1) (x-4). We can assume that the separation between the minus sign and the fraction led the student G49 to consider that this operation indicated by the minus sign is relative to the entire fraction. Thus, it emphasizes visually the fraction, in detriment of the binomial x-4. Perhaps there had been a mechanic learning of distributive property in Elementary School and the student remained a scheme as in Figure 1, but without understanding their meaning. As the student did not recognize that $-\frac{x-4}{2}$ indicates $(-1)\left(\frac{x-4}{2}\right)$, the student shows that did not recognize the possible operations, so that, does not have the structure sense.

In the example of Class II, we observed that the main error of this resolution appears in the second member, when the student G33 multiplies the integer number 2 by the denominator 4, obtaining a denominator equal to 8. G33 seems to have formed a concept image to reduce to the same denominator that requires the multiplication of the denominator for each numerator, without realizing that number 2 is an integer, equivalent to rational $\frac{16}{8}$.

In the example of Class IV, despite presenting correctly the first steps of the solution, the student M40 does not know what to do with the resulting equation, perhaps because the denominators are not equal on both sides. It lacks, perhaps, the structure sense, because it does not recognize the manipulations that could do to continue the resolution.

In Class V, student I2´s answer is interesting because it shows that he wrongly assumed that the denominator of the fraction of the second member could multiply the entire first member. It seems that this student has a mistaken concept image about the "cross multiplication".

Proposal to create a technological resource

Due to the large percentage of errors categorized in class I, there is concern about understanding and preparing some resources to help the students to overcome these difficulties. After all, the students are freshmen in Differential Calculus and algebraic manipulation is a necessary condition to work with limits and derivatives. Face the difficulties regarding the content of the Elementary School but often found in higher education students' answers, it is suggested the use of some technological resource to assist the teacher to detect the errors and the students realize their difficulties. In the case of mathematics, when we mention technology to support the teaching and learning processes, it is quite common the reference to Dynamic Geometry Systems (DGS) and Computer Algebra Systems (CAS). In fact, the use of Dynamic Geometry Software in math classes is enriching, since enables experimentation, allowing a reversal of the traditional order of teaching, starting with research and then leaving for theorizing (Borba & Penteado, 2007). Experimentation is a key step in the knowledge construction, because it stimulates research. Furthermore, through the use of software, it is possible to promote interactivity, creating rich environments for learning.

Besides the Dynamic Geometry software, Learning Objects (LOs) have been widely used nowadays, as an essential resource for classes. The definition of LO is given by several authors, and may have some differences, but with a common base. McGreal (2004, p. 21) comments that "LOs are sometimes defined as being educational resources that can be employed in technology supported learning." More specifically, a LO can be reused a number of times in different learnings and can "combine text, images and other media or applications to deliver complete experiences, such as a complete instructional event." (Wiley, 2000, p. 7).

In this perspective, it is very convenient create a LO which is able to assist the student in the difficulties encountered in the study presented. This object, in turn, should contain characteristics of multimodality. According to Roswell and Walsh (2011, p. 55-56), "Multimodality is the field that takes account of how individuals make meaning with different kinds of modes." In the case of this proposal, LO should simultaneously use two modes of presentation: verbal and nonverbal. Its base should be the distributive property, given the errors found in the issues discussed. However, this property will be addressed generically, applied to many operations in which it is valid, not only addition and multiplication (as, for example, conjunction and disjunction in Logic, as well as union and intersection in Set Theory).

In the current phase of the research reported here, the object is under construction, both the content and the implementation of technological resources needed. It is called "Real Numbers: operations and properties", since that, to the distributive property be worked, it is necessary to pass by the other properties of addition and multiplication, paying attention to the fact that the distributive property "connect" both operations in the set in question. In the introduction, there will be the student's awareness about the importance of the study of real numbers, operations and properties, since this will be his universe of work in Differential Calculus. After this step, we proceed to a brief interactive, which presents the blocks of the object. At this point, it is suggested a navigation order, through a path created with arrows, but leaves it clear to the student that if he deems appropriate, may visit only one of the blocks, change the order or return a block already considered. The steps (or blocks) are organized as follows:

1. Test your proficiency level: at this stage, besides exercises of addition and multiplication with integer and rational numbers, as well as roots, it will be explored the meaning of some words in the context involved, as commute, associate, distribute, and so on. If the student does not have a good performance, will be given materials that can serve as reinforcement to these studies and obtainment of the necessary prerequisites for the study of the content.

2. Addition and multiplication of Real Numbers: this block will present a historic part on the addition and multiplication, as illustration, since the formal definition of operation is not the object of study in Calculus courses.

3. Properties of addition and multiplication: with respect to the properties, there will be a brief review of the specific properties of the operations involved, such as associative, commutative, neutral, and so on.

4. The distributive property: this is the "key block" of the object, in which will be studied the distributive property in several instances in which it appears, with theory and examples.

5. Extending the idea: this time, the concepts presented will be extended to other sets and situations. As previously mentioned, will be used, for example, sets and propositions.

6. Exercises: this is the stage of testing the knowledge acquired, in which the student can verify if he or she understood what was presented and back to the theory if it is appropriate.

Finally, it is emphasized that, in order to ensure the best educational use of the object, all screens should be allowed to control the pace by the user, which can pause, go back or change the navigation order as it deems most appropriate, although it is suggested a logical order. At all times, there will be a combination of visual and additives stimulus, as well as the use of verbal and nonverbal modes, avoiding, however, to overload cognitively the learner.

Final remarks

In this article, we present a report of a survey that aimed to analyze learning difficulties presented by students of Differential Calculus and possibilities to overcome these difficulties by technological resources.

We sought to support the analysis of mistakes in ideas of authors who have worked with the teaching and learning of algebra. To create a technological resource that can help students overcome their difficulties, it was planned to build a Learning Object with features mentioned in the literature and based on the highest difficulty, related to the distributive property of multiplication over addiction.

It is believed that with the creation of the LO through the difficulties presented by the students, it will be possible to contribute to other students also entering in courses which have mathematics in their curricula; they can expand their knowledge about properties of operations with real numbers. In this aspect, the errors the students could undertake in studies of Differential and Integral Calculus involving this base could be overcome.

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