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Abstract

Gifted students have potential to improve countries and this potential can be revealed and developed in schools where they spend most of their times with other regular students. However, these classrooms have some limitations for them; hence, they need some differentiated activities. Usage of dynamic geometry in mathematics lessons could be an opportunity to provide differentiated activities. Therefore, the aim of this study is to explore effects of mathematics lessons integrated with dynamic geometry activities on students' van Hiele geometric thinking levels controlling their probability for mathematical giftedness. Participants of the study were fifty-three fifth graders from a private school in Marmaris, Turkey. These students were grouped in terms of their probability of mathematical giftedness. Seven dynamic geometry activities about properties of line segments, triangles and quadrilaterals were developed and implemented in classrooms with tablets. Van Hiele geometric thinking level test was administered to the participants as pre-test and post-test. Results showed that dynamic geometry activities help students to move from geometric thinking level about recognizing shapes with visual clues to higher level about geometrical properties of shapes, namely relationship among shapes and their properties. Moreover, interaction between their probability for mathematical giftedness and improvements in geometric thinking levels were found. This study may contribute both to the gifted education and mathematics education fields by exploring the improvements in geometric thinking level and differentiated opportunities for gifted students. As a suggestion, a more comprehensive experimental study with larger samples so as to obtain generalize the findings could be conducted as a further study.

Key words: Mathematically gifted, Mathematics education, Dynamic geometry activities, Geometric thinking levels.

Introduction

Recently, gifted students are seen as having the potential to improve countries with the help of their ability to solve problems in creative ways as well as their property of leadership (Hannah, James, Montelle & Nokes, 2011; Maryland, 1973). According to National Association for Gifted Children (NAGC, 2005) gifted students "shows or has the potential for showing, an exceptional level of performance in one or more areas of expression" (p. 4) and this property makes them important for the future of the countries. Therefore, developmental and educational needs of gifted students were seen as the vital points that should be given necessary importance in educational environments. Likewise, most countries accepted the issue about enhancing gifted students' potential as their social requirement (Trna, 2014). Giftedness of children was thought to be related not only with intelligence or IQ scores of relevant tests but also with creativity and other various factors. Three Ring Conception of Giftedness Model of Renzulli (1979) addressed students' ability levels, their motivations on learning phase and their creativity as some factors related with giftedness of children. In line with this, Sternberg (1997) mentioned about high levels in analytical, creative and practical facets as factors affected giftedness in his Triarchic Theory. Some researchers also referred to some characteristics to identify giftedness of children. For instance, Davis and Rimm (2004) referred to high level of success and superiority in language usage, quick learning with enjoyment, memory retention, problem solving ability, reasonable curiosity and high level of thinking, attention levels in tasks and interests to contents as indicators for giftedness of children. Moreover, Baykoç (2014) stated that giftedness could exist as a capacity related to genetics as well as

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being related with environmental factors. Therefore, giftedness could be developed and trained within appropriate environment.

Mathematical Gifted Students and Gifted Education

In some research, giftedness of a child is referred as giftedness on specific domain or on any domain (Mayer, 2005). Therefore, giftedness on mathematics could be considered as one of the specific domains of giftedness. Among gifted students, mathematical gifted students are the ones who could see the world from the eyes of mathematics (Krutetski, 1976) and differ from other students in the abilities like spontaneous formation of problems, flexibility in handling data and originality in interpretation (Greenes, 1981). Although there is not still a common definition for mathematical giftedness (Pitta-Pantazi, Christou, Kontoyianni & Kattou, 2011), some studies mentioned that mathematical gifted children could organize tasks, use new statements in patterns and study advanced concepts better than regular children (Davaslıgil, 2004; Miller, 1990). Moreover, Johnson (2000) stated that examining quality of thinking about mathematical reasoning is the other way of determining mathematical giftedness. From another perspective, it is a misconception that conducting calculations does not directly and solely indicate mathematical giftedness; in fact, comprehension of mathematical ideas is the indicator of mathematical giftedness (Karaduman, 2010). Similarly, Sheffield (1994) defined features of mathematical giftedness as speed of understanding, higher level of ability in questioning, and seeing cause-effect relations in mathematical constructs. Mathematical gifted children differ from regular children for features of learning speed, understanding levels and interest levels to mathematics (Dağlıoğlu, 2004). Furthermore, mathematical creativity is also referred as another feature for differentiating mathematical gifted children by many other authors (Leikin, 2009; Sriraman, Haavold & Lee, 2013).

Apart from how they are defined, it is a reality that gifted students spend their times in the school with other regular students (Baykoç, 2011). Although these regular classroom environments are required for their social and emotional needs, these environments have some limitations in terms of their cognitive needs (Baykoç, 2014). When the lessons were not appropriate to their pace of learning and differentiated abilities and interests, the students face with boredom, lack of enjoyment or negative disposition towards mathematics (Maxwell, 2001; Park & Park, 2006). Thus, mathematical gifted students need differentiated activities that meet their differentiated needs, support their interest and ability domains, and motivate them toward mathematics tasks in classrooms (Baykoç, 2010; Pierce et al., 2011). As in the case in other countries, gifted students in Turkey need special activities or services that are outside the needs of regular students (Aydemir & Çakıroğlu, 2013). There are some institutions that support gifted students in Turkey. For example, Science and Art Centers (BİLSEM) under the auspices of MoNE and children universities as well as some private institution/centers for gifted children try to provide opportunities for gifted students in the remaining time from their school hours. There is a great need for learning and meeting their needs both in schools and in these institutions where they have only opportunity to be nurtured. Thus, opportunities to be offered to these students in any environments should be increased and enriched by means of research based studies. Based on this need, some studies related with mathematical gifted students concentrated on the modifications, differentiation or inclusion on the math curriculum/instruction to analyze their effects on students' achievement and to better fit the requirement of gifted students (Gavin, Casa, Adelson, Carroll, Sheffield & Spinelli, 2007; Thomas, 2019; Tieso 2003; Ysseldyke, Tardrew, Betts, Thill, & Hannigan, 2004). Additionally, some studies (Deringöl & Davaslıgil, 2020; Erdogan & Yemenli, 2019; Kamarudin, Kamarulzaman, & Ishak, 2018) examined mathematically gifted students' views and attitudes. For example, Hammer (2002) explored precocious mathematics students' attitudes when they were not challenged appropriately. Likely, Martin and Pickett (2013) mentioned their implementation of differentiated instruction and its effects on the improvements of the mathematical gifted students' motivation and engagement. However, as Ysseldyke et al. (2004) indicates, studies in literature reveal the problem that most gifted students deprive of learning environments which enables them to construct their own learning. Since gifted students have differentiated needs in line with their differentiated properties, learning tasks provided to mathematical gifted students should support differentiated characteristics like challenging, entertaining and so on (Özdemir, 2018). At that point, technology integration is one of these characteristics that both helps to construct and lead their own learning and a dimension suggested for the differentiated needs of mathematically gifted students (Özdemir, 2016).

Geometry Education and Technology

Geometry is one of the fields of school mathematics. Since our environment, with which we are surrounded consists of many geometric shapes and objects, geometry education has its own importance. Geometry can be considered as an important skill of doing mathematics (Suydam, 1985) and geometry education provides opportunities to enhance logical thinking abilities, spatial insight about physical environment and knowledge for understanding higher level mathematics (Suydam, 1985). Literature review revealed that the van Hiele geometric thinking hierarchy has commonly been considered to describe learners' knowledge and thinking about two-dimensional geometry (Battista, 2002; Olkun, Sinoplu & Deryakulu, 2005; Özçakır & Çakıroğlu, 2019). Van Hiele explained factors related to enhancing logical thinking, spatial insight and understanding higher level of geometry within van Hiele geometric thinking hierarchy (Usiskin, 1982). In this hierarchy, geometric concepts were arranged into levels in accordance with prerequisite concepts suitable to students' geometric thinking. This theory of geometric thinking is consisted of five levels as visualization, analysis, informal deduction, formal deduction and rigor (Crowley, 1987). The geometric thinking levels within this hierarchy are organized as;

- Level 0 – Visualization: visual clues about geometric figures, recognizing figures depends on visual information about figure.
- Level 1 – Analysis: geometric properties have their own value, recognizing includes simple definition and properties of figure.
- Level 2 – Informal Deduction: properties can form a family of figures, interrelationships between figures based on their similar or different properties.
- Level 3 – Deduction: going beyond identifying properties of figures and relationships among them, proofs can be constructed, using postulates or axioms and definitions.
- Level 4 – Rigor: learner could go beyond Euclidean geometry and can work in different geometric and axiomatic systems.

National Council of Teachers of Mathematics (NCTM, 2010) suggests that students should proceed first level of van Hiele geometric thinking hierarchy at kindergarten to second grade, second level at third grade to fifth grade and third level before graduated from middle school. Hence, it was suggested that students should have achieved first three levels of this geometric thinking hierarchy at middle school in order to understand high school mathematical concepts (Cansız-Aktaş & Aktaş, 2012). However, progresses between these geometric thinking levels are related with educational experiences of learners rather than their ages or maturations. Therefore, teaching geometry with experiment-based activities could foster students' understanding in geometry (Fidan & Türnüklü, 2010; Özçakır, 2013; Tan-Şişman & Aksu, 2012). Moreover, students can learn geometric concepts sufficiently when learning environments are prepared in line with their knowledge on geometry and their geometric thinking levels (Choi-Koh, 1999). Therefore, understanding learners' knowledge and linking with van Hiele hierarchy is important for developing suitable learning activities, materials and so instructions, to provide them learning environments in which they could advance through the levels of van Hiele with these learning opportunities (Malloy, 2002).

Computer technology can provide these learning opportunities for learners since educational technology can be helpful to provide tasks and tools included with multiple representations of concepts dynamically linked together. It also offers learning environment with different opportunities to concretize an abstract concept of mathematics with digital dynamic contents and virtual objects (Özçakır, 2013). Therefore, these tools provide students a digital environment for exploring and identifying mathematical concepts and relationships within or among mathematical objects (Thomas & Holton, 2003). Dynamic geometry software is one of the technological tools used in mathematics education. Dynamic geometry software permits students to interact with mathematical constructs so they can examine different examples of the constructs with dynamic features. In other words, students have an opportunity to investigate mathematical objects like in a laboratory for mathematics (Tabach, 2011). In this environment, students can manipulate dynamic geometric objects and observe changes in multiple representation of the objects provided by hot links among these representation and real-time measures (Laborde, Kynigos & Strasser, 2006). Therefore, they can make tests and observe changed and unchanged parts of the objects among various manipulation, can record and conjecture constructs and theorems with dynamic geometry software.

Although, in various research, dynamic geometry activities are found to engage students' understanding of geometry and to provide a learning environment to foster their geometric thinking (Gawlick, 2005; Karakuş & Peker, 2015; Özçakır, 2013), there still needs to understand effects of dynamic geometry activities on both gifted and regular students' geometric thinking levels. Since technology enables the students to continue with their own learning pace (Özçakır, 2013), it may be seen as a useful way for mathematical gifted students to meet their needs in classrooms (Johnson, 2000; Periathiruvadi & Rinn, 2012). Based on this idea, in instructional phase, using educational technology for learning tasks could be an opportunity to develop abilities of mathematical gifted students. In order to increase the motivation of mathematical gifted students in lessons, usage of dynamic geometry can be seen as a valuable opportunity. It also enables students to examine the geometric concepts with details as one of the needs of gifted students in mathematics classrooms. Furthermore, this may help them to use their giftedness potential and proceed with their own pace. Therefore, this study aimed to explore effects of mathematics lessons integrated with dynamic geometry activities on students' van Hiele geometric thinking levels controlling their probability for mathematical giftedness.

When studies (Levenberg & Shaham, 2014; Shillor, 1997, Taylor, 2008) about gifted and geometry education analysed, the lack, importance and the need in this area can be seen. For example, Tanahan's (2006) study with high school geometry teachers across California highlights that although teachers in high school geometry understand the need of differentiated education for gifted students, they do not significantly differentiate their instruction. Besides, Casa et al. (2017) indicate the need for more challenge and special activities for the students in kindergarten. A similar study to this study conducted by El-Demerdash (2010), the effectiveness of an enrichment program using dynamic geometry software on developing mathematically gifted high school students' geometric creativity can also be seen as an indication for such kind of studies in also middle school level. Moreover, some researchers (Johnson, 2000; Siegle, 2004) mention about the role of technology integration in gifted education due to the ability and motivation of gifted students in technology usage. Even, Kontostavlou and Drigas (2019) mentioned about proved effectiveness of technology usage in special education and provided a report about studies related with using technology for gifted education. That is, because technology allows to proceed at their own pace (Kaput, 1992; Özçakır, 2013), technology integration can be seen as one of the valuable characteristics for the task of mathematically gifted students (Özdemir, 2016).

Method

Research Design

This study was designed as pre-test – post-test experimental research methodology without control group in accordance with the aim of this study and regularities of the school of the students. This method enabled the researchers to compare students' pre- and post-scores as well as their giftedness score. Since this school, which we had conveniently, did not allow us to administrate different instructional methods for student groups, there was no control group and all student groups learnt mathematics using dynamic geometry software within our experimental study. This study focused on exploring effects of mathematics lessons integrated with dynamic geometry activities on regular and mathematical gifted students' geometric thinking. Seven dynamic geometry activities about 5th grade mathematics objectives were developed, and students were engaged with dynamic geometry activities in lessons.

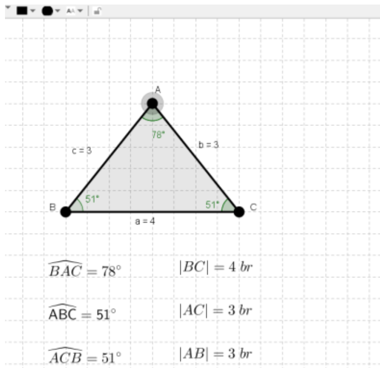
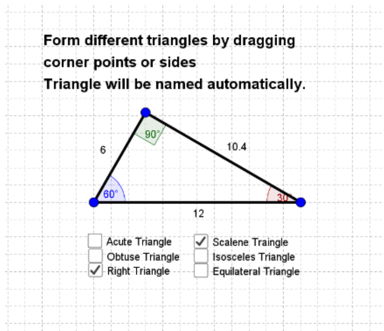
Participants

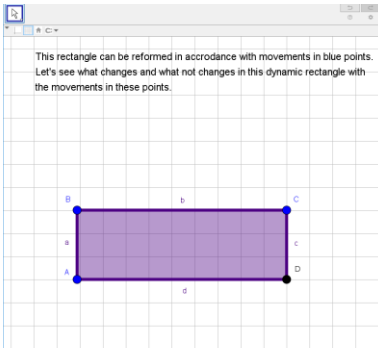
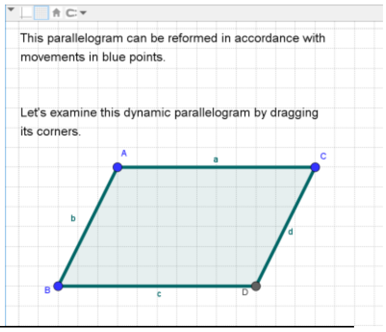
In this study, one of the researchers was mathematics teacher of the three classes of students and so convenient sampling was used, which formed our working group. That is, twenty-one female and thirty-three male 5^{th} grade students from three classrooms of a school in Marmaris, Turkey involved in this study. However, since this school only allowed similar treatment for all classes in the same grade level, all students participated in the treatment. Therefore, forming a control group was not possible in the context of this study. These students had generally experienced using tablets for educational purposes and so they had some background for technology supported learning. The participants were grouped for data analysis in terms of their probability of mathematical giftedness by using Test of Mathematical Abilities for Gifted Students (TOMAGS) (Ryser & Johnsen, 1998). Students' scores in the TOMAGS were used to form three different mathematically giftedness levels as low, average and high probability of mathematical giftedness levels. According to these scores, 25 students were grouped in low probability group, 11 students were grouped in average probability group and 17 students were grouped in high probability of giftedness group.

Research Procedure

In this study, seven dynamic geometry activities were developed in accordance with Middle School Mathematics Curriculum for 5th grade (Ministry of National Education [MoNE], 2018). The activities were about concepts related with properties of line and line segments, and properties and types of triangle and quadrilaterals such as rectangle, parallelogram, rhombus and trapezoid. While designing and implementing these activities, GeoGebra dynamic geometry software was used as a tool for learning since this dynamic geometry software supports multilanguage included Turkish Language and runs on multiplatform included mobile devices like tablet, PCs and mobile phones. The activities were designed as in line with learning objectives in the curriculum (MoNE, 2018) and as easy as possible to use GeoGebra as a learning tool via tablets. Since all needed figures provided to students, they did not have to construct any geometric figures in these activities. They only moved or relocated the points and line segments via touching and dragging in these activities. Moreover, in order to design suitable activities for students in different levels of van Hiele hierarchy, activities were designed to allow students to manipulate geometric figures dynamically by preserving their basic properties so that they have an opportunity to observe changed and unchanged constructs within figures. These activities were evaluated for their appropriateness by four researchers with doctoral degree in the field of Elementary Mathematics Education and two elementary mathematics teachers. According to their feedbacks, some changes in design were made and the activities were made ready for administration stage. Before administrating the activities to the classroom, a pilot study was conducted with two, 6th and 5th grade, students from a different school and the points and instructions that need to be clarified were reorganized to take the last form of the activities, some of which were briefly described in Table 1. That is, through the study, these dynamic geometry activities were used in mathematics lessons, which were designed by the researchers via GeoGebra dynamic geometry software.

Table 1: Some of the dynamic geometry activities designed in this study

Concept	Description	Sample images
Line segments	There were several line segments in four different positions and lengths. For each line segment one can move or drag points of an orange line segment to make parallel and in same length to other black one. If one successfully completed task dynamic geometry activity confirms results with a green tick.	
Triangle - Properties	Three dragging points to explore angle – side relationships for a triangle. One can stretch sides by dragging points and observe changes in angle measures and side lengths.	
Triangle - Types	There are three dragging points to explore properties of types of triangles. One can drag corners or sides to change appearance of the triangle and dynamic geometry activity automatically named triangle type.	

Rectangle	There are three dragging points to explore a rectangle: A – rotate rectangle B – stretch rectangle vertically C – stretch rectangle horizontally One could do some measurements to examine the rectangle and extract its properties.	
Parallelogram	There are three dragging points to explore a parallelogram: A – change lengths of a and c sides B – stretch parallelogram vertically or horizontally C – change length of b and d sides One could do some measurements to examine the parallelogram and extract its properties.	

During the study, at first, an hour preparation course for using GeoGebra on tablet PCs was introduced to these students. Moreover, the first activity, which is about line segments, was used to make students familiar with the usage of GeoGebra in learning environments. Then, the dynamic geometry activities were implemented during six-hour lessons with tablet PCs in classrooms. Since one of the researchers was teacher of these students, this researcher had full control of teaching processes during the study. In these activities, students did not have any technical difficulties, so they easily used GeoGebra as a learning tool. In the learning process, they were observed as active participants in learning environment, and they allowed to share ideas about their explorations freely. In general, students dealt with learning activities and GeoGebra as well as sharing ideas and engaging discussions throughout the research. In the meanwhile, the teacher who was also one of the researchers, guided students if they struggle in both using tablet and engaging learning activities, while monitoring students' works and giving feedback on their progress.

Instruments

In this study, a test to determine students' probability of mathematical giftedness, which was TOMAGS, was administered to all students at the beginning of the study. Items in TOMAGS aim to identify children who are gifted in mathematics (Ryser & Johnsen, 1998). Therefore, the TOMAGS could be used as an identification instrument for mathematical giftedness due to its strong validity and reliability scores, as greater than .80 which can be seen in Table 2 (Ryser & Johnsen, 1998).

Table 2. TOMAGS validity and reliability scores (Ryser & Johnsen, 1998, p. 28).

	Content Sampling		Time sampling	Scorer	Average
	Normal	Gifted			
TOMAGS Intermediate	0,88	0,86	0,94	0,99	0,93

The TOMAGS includes 47 open-ended questions with a degree of difficulty to test the limits of students in a problem-solving environment. These questions cover learning domains of middle school mathematics curriculum such as numbers, geometry, measurement, and statistics and probability (Ryser & Johnsen, 1998). Moreover, three curriculum standards of NCTM were represented in construction of TOMAGS such as mathematical problem solving, mathematical communication and mathematical reasoning. In the context for this research, these standards were also applicable since they are aligned with basic skills for mathematics defined in mathematics curriculum of Turkey (MoNE, 2013). Results of the TOMAGS demonstrated high reliability in all three types of error because reliability coefficients approximating or exceeding .80 are found reliable and .90 or above found as most desirable (Ryser & Johnsen, 1998; Salvia & Ysseldyke, 1995). Similarly, in order to evaluate the reliability and validity of the Turkish version of TOMAGS, Özdemir (2016) evaluated the test and concluded to the appropriateness of the adapted version of the test to determine mathematically giftedness of students in Turkey.

In this study, while identifying probability of mathematical giftedness, students' quotient scores were calculated based on their raw scores obtained in the test as well as their age in terms of month and year. Then, based on these quotient scores, probability of mathematical giftedness was determined in line with the guidelines of Ryser and Johnsen (Ryser & Johnsen, 1998, p.17) as low, average and high probability of mathematical giftedness within the context of this study. Thus, the participants of the study who are 21 girls and 32 boys, were grouped in terms of their scores in the TOMAGS. That is, among those fifty-three students, seventeen students were identified as high probability of mathematical giftedness, eleven students were identified as average probability of mathematical giftedness and remaining twenty-five students were identified as low probability of mathematical giftedness as seen in Table 3.

Table 3. Probability of giftedness for participants

Gender	Probability of Mathematical Giftedness			Total
	Low	Average	High	
Female	11	3	7	21
Male	14	8	10	32
Total	25	11	17	53

The participants were also pre-tested at the beginning of the study and post-tested at the end of the study by Van Hiele geometric thinking level test, which was developed by Usiskin (1982) to define students' van Hiele geometric thinking levels. This test was translated into Turkish by Duatepe (2004). The Van Hiele geometric thinking level test is consisted of 25 multiple-choice items. In this test, each 5-item were related with van Hiele Geometric Thinking Levels, respectively. Therefore, in this study, first fifteen items were used since these three levels are related with the intended grade level in accordance with the suggestion of NCTM (Fidan & Türnüklü, 2010).

Data Analysis

Scoring of the Van Hiele geometric thinking level test is criterion-based which suggested by Usiskin (1982). Usiskin clarified that if a student correctly answered three of five questions for each level, this student is considered as achieved the related level. Thus, for this test, students were assigned a weighted sum score in the following manner in Table 4. According to this scoring method, for general assessment for van Hiele geometric thinking levels of the group mean value, the intervals for placement were considered such that 1,00 - 2,50 points for Level 0, 2,51 - 5,00 points for Level 1 and 5,01 - 7,00 points for Level 2.

Table 4. Scoring van Hiele Level Test

Score	Criteria
0 Point	If at most two of first five question are correct
1 Point	If three of first five questions are correct
2 Points	If three of second five questions are correct
3 Points	If three of third five questions are correct

Data collected through TOMAGS was used to determine probability of mathematical giftedness of students and so, according to this data from administration of TOMAGS three group for students were formed. These groups constituted independent variable for data analysis. On the other hand, data collected through pre-test and post-test administrations of the Van Hiele geometric thinking level test provided scores of students regarding their geometric thinking levels. This data formed dependent variables for data analysis. Since this study included three groups of students and their scores for the Van Hiele geometric thinking level test over time, 3x2 mixed model repeated measure analysis of variance (ANOVA) was considered as appropriate to analyse this data. Besides, the mixed model repeated measure ANOVA is powerful to reveal the main effect of intervention by disregarding groups and interaction effect between intervention and groups of students. Based on this, preliminary analyses were conducted, and it was seen that the data satisfied assumptions of the mixed model ANOVA. Therefore, the mean scores from the pre-test and post-test administration of the Van Hiele geometric thinking level test in terms of groups formed by probability of mathematical giftedness were analysed through mixed model repeated measure ANOVA.

Results

The data were handled, and findings were reported in terms of van Hiele geometric thinking levels of students and their probability of mathematical giftedness. First of all, fifth grade students' scores regarding van Hiele geometric thinking level have been presented in accordance with their pre-test and post-test results from Van Hiele geometric thinking level test in terms of probability for mathematical giftedness, as a starting point for data analysis in Table 5.

Table 5. Descriptive statistics regarding scores of students on van Hiele geometric thinking level test

Test	Giftedness Probability	N	M	SD
Pre-test	Low	25	1,48	0,87
(M=1,72; SD=0,97)	Average	11	2,27	1,01
	High	17	1,71	0,99
Post-test	Low	25	2,44	1,96
(M=3,91; SD=2,47)	Average	11	4,27	2,24
	High	17	5,82	1,88

Descriptive data on Table 5 showed that van Hiele geometric thinking scores of students changed positively after intervention, since before the intervention students generally were placed in Level 0 (M=1,72; SD=0,97), they were placed in Level 1 of the van Hiele hierarchy after study (M=3,90; SD=2,47). Students' placements for the van Hiele hierarchy in terms of their probability for mathematical giftedness changed also positively between pre-test and post-test administrations. In detail, at the beginning of the study, all students who have low probability of mathematical giftedness (M=1,48; SD=0,87), average probability of mathematical giftedness (M=2,27; SD=1,01) and high probability of mathematical giftedness (M=1,71; SD=0,99) were defined at Level 0 of this hierarchy. After students' completion of the dynamic geometry activities, their status of van Hiele hierarchy changed as in the following manner. That is, students with low probability were still categorized as at Level 0 (M=2,44; SD=1,96), while students with average probability were categorized as at Level 1 (M=4,27; SD=2,24). Additionally, students with high probability of mathematical giftedness were stated at Level 2 (M=5,82; SD=1,88) of this hierarchy.

In order to investigate effects of intervention with dynamic geometry activities on van Hiele geometric thinking levels of students in terms of their probability of mathematical giftedness, their pre-test and post-test scores on the Van Hiele geometric thinking level test were analysed through 3x2 mixed model repeated measure ANOVA (Table 6).

Table 6. Mixed model ANOVA results for main effect and interaction

Source of Variance	SS	df	F	p
Tests	132,010	1	105,841	0,000
Tests * Giftedness	50,694	2	20,322	0,000
Error	62,362	50		

The output of this mixed model ANOVA analysis included two main results about the data. First of all, according to mixed model repeated measure ANOVA results, there was a significant main effect of intervention with dynamic geometry activities on students' gains about geometric thinking levels between pre-test and post-test results of Van Hiele geometric thinking level test ($F(1, 50)=105.84$, $p<0.05$). That is, in a general manner, intervention with dynamic geometry activities about geometry concepts had a significant effect on students' van Hiele geometric thinking levels even if we ignore their probability of mathematical giftedness (Table 7).

Table 7. Distribution of van Hiele Levels for main effect

Test	M	95% CI	Frequencies for levels		
			Level 0	Level 1	Level 2
Pretest	1,72	1,45 - 1,98	34 (64%)	19 (36%)	0 (0%)
Posttest	3,91	3,23 - 4,59	14 (26%)	20 (38%)	19 (36%)

In the Table 7, it was revealed that, at the beginning of the study, most of the students were at Level 0 of van Hiele geometric thinking hierarchy regarding mean of their pre-test scores (M=1.72, SD=0.97) and they were able to reach Level 1 of van Hiele hierarchy at the end of the study according to mean of their scores in post-test administration of the Van Hiele geometric thinking level test (M=3.91, SD=2.47). Therefore, this main effect implies that the intervention with dynamic geometry activities could help students to move on from visualization level of van Hiele geometric thinking hierarchy to analysis level and also, even for some students, to informal deduction level as seen on the Table 7.

Furthermore, according to mixed model repeated measure ANOVA results on Table 6, there was also a significant interaction between probability of giftedness in mathematics of students and differences in their scores from pre-test and post-test administration of the Van Hiele geometric thinking level test ($F=2, 50)=20.322$, $p<0,05$). This interaction effect signifies that students' probability of mathematical giftedness had influences on their gains from the intervention with dynamic geometry activities about geometry concepts in terms of van Hiele geometric thinking levels (Figure 1).

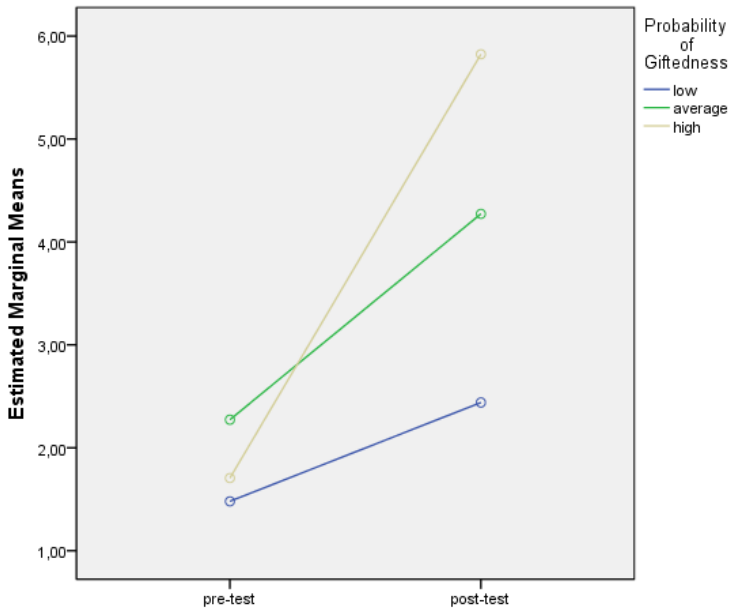


Figure 1. Estimated marginal means for van Hiele geometric thinking levels test

Students' placements for van Hiele geometric thinking hierarchy changed incrementally regarding their probability for giftedness in mathematics between pre-test and post-test administration of the Van Hiele geometric thinking level test. In detail, mean scores for Van Hiele geometric thinking level test administrations of low probability of mathematical giftedness students changed from 1,48 (SD=0.87) to 2,44 (SD=1.96), scores of students with average probability changed from 2,27 (SD=1.01) to 4,27 (SD=2.24) while scores of students with high probability changed from 1,71 (SD=0.99) to 5,82 (SD=1.88) due to intervention for dynamic geometry activities about geometry concepts. Hence, the estimated marginal means expressed nature of the interaction between students' probability of giftedness and their scores in Van Hiele geometric thinking level test administrations. The graph in Figure 1 clearly shows that all three groups of students benefited from the intervention positively. However, students with low probability of mathematical giftedness benefited from intervention at minimum level while students with high probability benefited from intervention at maximum level in terms of their scores in Van Hiele geometric thinking level test. Additionally, students with higher probability of mathematical giftedness performed more incremental progress from other groups of students (Table 8).

Table 8. Distribution of van Hiele Levels for main effect

Giftedness Probability	Test	M	Frequencies for levels		
			Level 0	Level 1	Level 2
Low	Pretest	1,48	19 (76%)	6 (24%)	0 (0%)
	Posttest	2,44	13 (52%)	9 (36%)	3 (12%)
Average	Pretest	2,27	4 (36%)	7 (64%)	0 (0%)
	Posttest	4,27	1 (9%)	6 (55%)	4 (36%)
High	Pretest	1,71	11 (65%)	6 (35%)	0 (0%)
	Posttest	5,82	0 (0%)	5 (29%)	12 (71%)

According to the interaction effect, intervention with dynamic geometry activities could help students to move between levels of van Hiele hierarchy but differently in terms of their probability of giftedness in mathematics as described on Table 6. In all groups, some students accomplished to reach level 2 of van Hiele geometric thinking hierarchy but in different rates. 12% of students in low probability of giftedness group places at level 2, while 36% of students in average probability of giftedness group and 71% of students in high probability of giftedness groups places at this level.

Discussion

In this study, examining the effects of mathematics lessons integrated with dynamic geometry activities on students' van Hiele geometric thinking levels controlling their probability for mathematical giftedness was aimed. The results of the study enlightened the effects of mathematics lessons supported by dynamic geometry activities on geometric thinking as well as interactions among students' probability of mathematical giftedness and their gains for geometric thinking.

First of all, dynamic geometry activities in this study helped to move forward in levels of van Hiele hierarchy for fifth grade students. Many studies stated that dynamic geometry supported learning activities have great influences on geometric thinking (Karakuş & Peker, 2015; Özçakır, 2013). According to results, mathematics lessons supported by dynamic geometry activities helped students to make transitions from visualization level to analysis level of geometric thinking hierarchy regarding difference on mean scores from pre-test and post-test administration of Van Hiele geometric thinking level test. The main effect of intervention implied that all students benefited from education supported with dynamic geometry activities if we disregard their probability of mathematical giftedness and consider them as one group of students. Since these dynamic geometry activities provide students a dynamic learning environment to explore mathematical constructs like doing experiments for mathematics (Gawlick, 2005), their geometric thinking levels improved. This dynamic feature of the activities allows students to make experiments for mathematics by dragging, making manipulations, exploring quadrilaterals in different positions, and realizing unchanged and changed properties in these movements (Fidan & Türnüklü, 2010; Özçakır, 2013; Selçik & Bilgici, 2011). Therefore, dynamic feature of dynamic geometry activities gave students an opportunity of working with dynamic figures and so inspecting the same concept with numerous different drawings because a dynamic figure always preserves its basic properties and make these basic properties solid. In other words, with dynamic figures, a student can change the figure easily while maintaining its basic features. Therefore, students in this study, had access to all of these possible variations of triangle, rectangle, parallelogram, rhombus and trapezoid.

The other crucial point highlighted in the findings of the study is that students having high probability of mathematical giftedness were moved from visualization level to higher level of geometric thinking. Most of these students were in informal deduction level of van Hiele hierarchy at the end of the study that when compared to the low or average probability of students. In other words, most of the students who could reach to the informal deduction level at the end of the study were students having high probability of giftedness in mathematics. These findings coincided with the idea that mathematical gifted students require differentiated materials that are not restricted only with the curriculum requirements and the tasks that they can discover the ideas (Johnson, 2000). Moreover, as stated by Özdemir's (2016) study, technology integrated tasks, may enable them to follow with their own speed and provides an individualized pathway to them. As seen in the findings of the study, other students could only move at most one level, mathematical gifted students could go further. This can be interpreted in this way that such technology integrated tasks might be a good way to meet the differentiated needs of mathematical gifted students (Baykoç, 2010; Pierce et al., 2011). Furthermore, when it was seen in the Vygotsky's (1980) point of view, with regular classroom activities, mathematical gifted students have to follow other students' zone of proximal development by proceeding at the same pace with others. However, it is also essential that they are scaffolded in their own zone of proximal development to move further in mathematical concepts. Additionally, these findings also coincide with the Edwards's (2006) idea that, geogebra may be seen an effective tool for gifted students' materials due to its easiness in planning of enrichment activities. Similarly, El-Demerdash (2010) concluded that enrichment programs supported by dynamic geometry activities has a positive effect on mathematically gifted students' geometric creativity. Moreover, the other crucial point revealed from the analysis was such that the mean score of students having high probability of giftedness was not so high in the beginning of the study while they had a heavy increase at the end of the study. Even, the students who could reach to the level 2 was mostly high probability students. However, low level of these students' means scores when compared to other groups of students was remarkable in the pre-test scores. At that point, what needs to be discussed is the issue that with regular instruction and tasks, mathematical gifted students may get bored and they may not use their full potential (Johnson, 2000). However, by means of the study, they could find an interesting opportunity to go beyond in geometry.

Conclusion

Based on these, findings of the study could provide some valuable hints both to the area of theory and practice. Initially, providing students a learning environment, which includes dynamic geometry learning materials, gives them to access different variations of the same geometric figures thereby, explore mathematical constructs and make discoveries via dynamic interactions. This learning environment could help students to focus on properties of geometric constructs among different orientations of the same figures. Results of this study reflected that dynamic geometry activities helped students to move from shape properties to geometrical properties, namely relationship among shapes, and even from recognizing figures based on visualization to realizing relationship among shapes. Furthermore, the role of these activities on gifted students' performance is another issue that needs to be highlighted. Due to the fact that, both research and application area requires various examples that can be provided to gifted students, this study could be seen as an opportunity for this.

In conclusion, dynamic geometry activities are powerful medium for improving students' geometric thinking, especially for gifted students. Teachers can use activities consisted of dynamic figures while teaching quadrilaterals concepts or other many geometric concepts in order to engage their students in situations where they make mathematical experiments. Moreover, this study is limited to the participants of the present study that a more comprehensive similar study could be conducted with different samples so as to generalize the findings of the study. Moreover, a longitudinal and experimental study that can reveal the effects of such intervention to students' geometric thinking can be conducted for the further research study.

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