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Preservice Middle School Mathematics Teachers' Definitions of Algebraic Expression and Equation

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Abstract

Using correct definitions of the mathematical concepts is crucial for learning and teaching of any mathematical content. Being able to make mathematically correct definition of the concepts is an indicator of teachers' content knowledge. The purpose of this study is to determine how preservice middle school mathematics teachers define the concept of algebraic expression and equation. The participants of this case study were 35 preservice middle school mathematics teachers. The data were collected through written exam and semi-structured interviews. Written exam includes two questions asking preservice teachers to define equation and algebraic expression and write an example of each. Only 9 of participants correctly defined algebraic expression. Preservice teachers' definitions of algebraic expression were classified under three themes which are expressions containing unknown, expressions containing equality, and mathematical expressions. Two themes arose from preservice teachers' definition of equation: expressions with unknown, and expressions with equality.

Key words: Preservice mathematics teachers, mathematics teacher education, algebraic expression, equation.

Introduction

Teachers are the most important factors affecting teaching and learning process of mathematics (Ball, 1990; Charalambous, Hill, Chin & McGinn, 2019; Shulman, 1986, 1987). Teachers' knowledge about the subject that they will teach is the most important factor affecting student success. Content knowledge is the knowledge of teachers about definitions of concepts, relation between concepts and mathematical notation of subject they teach (Ball, Tames &d Phelps, 2008). For students to learn mathematics topic and concept correctly, teachers should have sufficient content knowledge about the subject (Ball, 1990, Ball, Thames & Phelps, 2008; Fennema, Sowder & Carpenter, 1999). Researches show that teachers' content knowledge has effect on their teaching practice and thus students learning (Ball, Lubienski & Mevborn, 2001; Brizuela, 2016; Charalambous et al., 2019; Copur-Gencturk, 2015; Hill & Ball, 2004; Hill, Rowan & Ball, 2005; Tchoshanov, Cruz, Huereca, Shakirova, Shakirova & Ibragimova, 2017). It cannot be expected from a teacher with lack of knowledge on subject he teaches to perform a successful teaching practice (Ball, 1990; Ball et al., 2008; Dreher, Lindmeier, Heinze & Niemand, 2018). To strengthen students' mathematical knowledge, teachers should be strengthened first (Ma, 2010).

Teachers should know the concepts included in content area and different representations of the relationships between these concepts (Ball, 1990). One of the most important dimensions of content knowledge is to be able to make a mathematically correct definition (Ball et al., 2008). Teachers should be able to define the concept correctly first. Making correct definition of concepts is the basis of concept knowledge about the subject that, they will teach (Ball, 1990, Ball et al., 2008).

Learning algebra is challenging for students and teachers' content knowledge related to mathematical concepts determine how students make sense of algebraic concepts (Stephens, Ellis, Blanton & Brizuela, 2017). Just focusing on operational properties of algebraic concepts prevents students from correctly structuring their mathematical meaning. Students develop many different misconceptions as they deal with concepts of algebra

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because they focus on rules and operation (Akkan, Baki & Çakıroğlu, 2012; Dede & Argün, 2003; Weinberg, Dresen & Slater, 2016). Students have difficulty to understand letter symbols used in algebra. They find it difficult to make sense of letter symbols as a numerical value that represents a variable (Brizuela, 2016; Christou & Vosniadou, 2012). Students also learn algebraic concepts incorrectly due to the erroneous meanings they attribute to letter symbols (placeholder, abbreviation of an object, etc.). For example, they define algebraic expression and equations as they include a letter and an operation symbol because of wrong learning related to letter symbol (Stephens et al., 2017).

The most difficult concepts in teaching algebra are the concepts of algebraic expression and equation. The concept of algebraic expression and equation form the basis of teaching advanced algebra (Wasserman, 2016). In order for these concepts to be learned correctly by students, teachers' content knowledge must be strong. Attorps (2003) stated that teachers' understanding of equation concept is incorrect. Teachers explained the meaning of the equation through operational processes. Another study examining conceptions of algebra conducted by Stephens (2008) included 30 preservice elementary teachers. Result of the study indicated that preservice teachers' content knowledge related to algebra is quite limited. Most of participants equated algebra just with manipulation of symbols and implied that solution strategies including traditional symbol manipulation may be more valuable for algebra learning than strategies demonstrating conceptual understanding. Stump and Bishop (2002) found that 30 prospective teachers who participated in their studies defined algebra with a focus on symbol, letter and problem solving, and they did not mention algebraic relations. Zuya (2017) worked with 54 preservice secondary school mathematics teachers to investigate preservice teachers' knowledge and skills related to algebra. Finding of the study showed that the preservice teachers' performances were very low in tasks demanding conceptual knowledge for solution while they performed above average in tasks requiring procedural knowledge. What do teachers view as algebra determines what will worth to teach in the classroom (Stephens, 2008). That is, teachers' definitions of algebra play an important role during organization of teaching process (Stephens, 2008; Stump & Bishop, 2002).

Teachers and pre-service teachers who view algebra not as a way of thinking but as a solution process in which a series of operations are performed do not use algebraic relations in operations involving basic concepts of algebra, such as algebraic expression and equation. They mostly focus on procedural information and cannot make conceptual explanations regarding these procedural processes (Agarwal, 2006; Black, 2007; Hohensee, 2017; Odumosu & Fisayi, 2018; Welder, 2007; Welder & Simonsen, 2011; Zuya, 2017). Therefore, it is important to determine how pre-service teachers define the two basic concepts of algebra; algebraic expression and equation. In this context, this study aims to determine how middle school mathematics teacher candidates define the concept of algebraic expression and equation, which form the basis of algebra learning.

Method

Case study design was adopted for this study. Case study is a qualitative inquiry which aims to explore a program, an activity, process or one or more cases in depth through multiple data sources (Creswell, 2007). Written exams and interviews were used as data sources. Case studies help researcher to investigate a problem in detail. Therefore, this study was conducted as a case study in order to investigate preservice middle school mathematics teachers' knowledge related to basic algebra concepts.

Participants

Participants of this study were 35 second grade preservice middle school mathematics teachers studying in a state university in Turkey. For selection of participants, convenience sampling and criterion-sampling methods were used. A convenience sample is a group of people who are chosen for a study because they are available (Fraenkel, Wallen & Hyun, 2012). , In convenience sampling, participants are selected from a group of people easy to reach in order to prevent loss of time, money and labour. (Büyüköztürk, Çakmak, Akgün, Karadeniz & Demirel, 2013). The criteria about determining the participant was to take "Introduction to Algebra" and "Linear Algebra" courses, which are important for improvement of preservice teachers' content knowledge of algebra. Taking and succeeding these course were taken as criteria because contents of these courses includes basic algebra concepts and proof of algebraic theorems. Preservice teacher learn background theorem of basic and simple algebra rules. So, at the end of these courses, preservice teachers should have deep understanding of algebraic concepts, theorems and logical explanation of operation with symbols. Also, preservice teachers' algebraic thinking skills are expected to develop after taking these courses.

Data Collection

Research data were collected through written exam and semi-structured interviews. In this context, the research was carried out in two stages. First, written exam that includes two open-ended questions was administered to preservice teachers to determine the definitions of the equation and algebraic expression. To ensure intelligibility of questions, written exam questions were checked and revised in terms of content and language by two experts and one in-service mathematics teacher. Preservice teachers were asked to define both equation and algebraic expression and write an example of each. To ensure reliability and validity, two experts checked content and language of questions. The written exam lasted 15-20 minutes. Second, clarification interviews were conducted with 7 preservice teachers. Interviewing is an effective way of checking accuracy of inferences that the researcher makes through written exam or observations (Fraenkel, Wallen & Hyun, 2012). Interviews help the researcher to understand what participants think. Interviewees were selected based on diversity of definitions. The interviews took on average 5 to 7 minutes. Interviews were recorded with an audio recorder.

Data Analysis

Content and discourse analysis were utilised to discover hidden meaning in preservice teachers' written and spoken expression. For content analysis, the processes of coding data, creating codes and themes, defining and interpreting the findings were followed (Patton, 2002). First, written exam papers of preservice teachers were examined and a summary of their answers to all questions were written up. Then, codes are established based on definition of preservice teachers. Lastly, codes were reviewed and general themes were created. Also, interviews were transcribed and data were analysed. To ensure reliability, all data were coded separately by two researchers. Then, codes and themes were discussed with the participation of third researcher and final version of codes and themes were created. The reliability rate between two coding was determined as 92%. Preservice teachers' definitions of algebraic expression led to creation of three major themes, which are expressions containing unknown, expressions containing equality, and mathematical expressions Also, preservice teachers' definitions of equation collected under two themes: expressions with equality and expressions with unknown.

Findings

Preservice Teachers' Definitions of Algebraic Expression

Preservice teachers' definitions of algebraic equation were collected under 3 different themes. Preservice teachers defined algebraic expressions as expressions containing unknown, expressions containing equality, and mathematical expressions (Table 1).

Table1. Themes related to definition of algebraic expression and number of participants

Theme	Categories	Number of participants
Expression containing unknown	Expression with unknown	12
	Expression with unknown, number and operation	9
	Expression with unknown and without equality	7
Expressions containing equality		4
Mathematical expression		3

Expression containing unknown: 28 of participating preservice middle school mathematics teachers defined algebraic expression as expression containing unknown. Three different codes were established*:* Expression with unknown, expression with unknown, number and operation, and expression with unknown and without equality. 12 of preservice teachers who defined algebraic equation as expression, which contain unknown refereed only unknown inclusion in their definition. These preservice teachers' definitions were erroneous. Some examples of preservice teachers' definitions are given below.

"It is an expression containing an unknown number."(7x-2)(2x+3y) (PT9)

"It is an expression of any number as unknown." (2x+y) (PT19)

Preservice teachers who defined algebraic expression as expression that contain unknown gave correct examples for algebraic expression. Although preservice teachers' examples of algebraic expression contain fixed terms and operations, they did not mention these concepts in their definitions. In the interviews, preservice teachers elaborated their definitions and stated that algebraic expression contains numbers and operations. During interview, participant 14 made following statement.

Researcher: *How would you explain algebraic expression?*

PT14: *For an expression to be an algebraic expression, it must contain an unknown. Actually, x, in 3x, is an algebraic expression own its own.*

Researcher: *What else can an algebraic expression contain?*

PT14: *Besides, there is another number in addition to unknown, 3+x is also an algebraic expression.*

Researcher: *So, what is important?*

PT14: *The important thing is that something performs an operation with the unknown.*

Nine of preservice teachers who defined algebraic expression as expressions containing unknown mentioned concepts of unknown, number and operation in their definitions. These preservice teachers defined algebraic expression correctly and gave correct examples. They also explained their definitions in detail during interview.

Seven of preservice teachers who defined the algebraic expression as expressions containing unknown stated that algebraic expression does not contain equality. Preservice teachers defined algebraic expression to a limited extent without equality. Explanation of Participant 18 during interview is given below as an example.

PT18: *It is an expression that involves at least one unknown. It is not expected this expression to be equal to a number (His gave (3a+4) as example of algebraic equation in written exam.).*

In the interviews, Participant 10 and Participant 18 emphasized that algebraic expression does not involve equality. It can be said that this situation is a result of preservice teachers' associating algebraic expression with the equation. The statements of Participant 18 reflect this situation.

Researcher: *You define algebraic expression as expressions with at least one unknown and it is not expected this expression to be equal to a number.*

PT18: *Yes, that is, there is equality in the equation. When equality is added to the algebraic expression, it becomes an equation.*

Researcher: *What is the difference between algebraic expression and equation?*

PT18: *As I said, there is no equality in algebraic expression. There is no value equal to.*

Expression containing equality: 4 of participants defined the algebraic expression as "expressions that contain equality" which is wrong. Below are some examples of their answers to the question in the written exam.

"It is expressions that we establish equality between numbers." (3x = 7) (PT22)

"They are numerical expressions containing equality." (2 + 3 = 5) (PT33)

The preservice teachers' examples for algebraic expression are also incorrect. Two preservice teachers gave an equation as an example of algebraic expression, and other two of them gave number sentence example. In the semi-structured interview conducted with Participant 6, the preservice teacher made the following explanation.

Researcher: *You gave 2x + 5 = 11 as an example for algebraic expression. Is 2x + 5 algebraic expression in this expression, or is this entire expression (2x+5=11) an algebraic expression?*

PT6: *Yes, whole of it is an algebraic expression.*

Mathematical expression: 3 of preservice teachers who participated in the study defined algebraic expression as *"mathematical expressions*" in written exams. 2 pre-service teachers defined algebraic expressions as they are "*mathematical expression",* while 1 preservice teacher defined algebraic expression as they are "*mathematical expression, something like rational and root numbers*". For algebraic expression, they gave 7/3, $\sqrt{5}$ and sin30 = 1/2 as examples. It was determined that algebraic expression definitions and examples of these pre-service teachers were wrong.

Preservice Teachers' Definitions of Equation

Equation definitions of the preservice teachers who participated in the study were collected under two different themes. In this context, preservice teachers defined the equation as expressions with equality and expressions with unknown. Name of themes and distribution of number of preservice teachers are given in Table 2.

Table 2. Themes related to definition of equation and number of participants

Theme	Category	Number of participants
Expression with equality	Expression with unknown	17
	Expression with algebraic expression	6
	Equality of two expression	4
Expression with unknown		8

Expression with equality: 27 preservice teachers defined the equation as "expressions that contain equality". Three different categories were obtained from definitions of preservice teachers: expression with unknown, expression with algebraic expressions and equality of two expressions.

17 of the teacher candidates who defined the equation as expressions with equality defined the equation as expressions with unknown. Preservice teachers used only concepts of equality and unknown in their definitions. In their definitions, they did not explain mathematical meaning of unknown inclusion in an equation. Examples of preservice teachers' definitions are given below.

"There is equality in the equation. It contains one or more unknowns." 2x + 3 = 11 (PT12)

"It is the equality of the unknown to a number or to an unknown." x + y = 3, x-y = 2 (PT1)

In the interviews, preservice teachers could not elaborate their definitions. On the other hand, examples given by preservice teachers in wren exam were correct examples of the equation. However, the preservice teachers gave examples of equations where the unknown is on the left side of the equation. In the interviews, it was seen that preservice teachers thought that the left side of the equal sign is the problem and it should be equal to a result. A part of interview conducted with Participant 16 in relation to this situation is presented as an example.

Researcher: *You explained the equation as an expression consisting of one or more unknowns, which is equal to a value. You have given examples of 3x + 6 = 20 and x = 2. Should there be a known value on the right side of equality?*

PT16: *Yes, so it has to be equal to a result. However, I am thinking now, there may be an unknown expression on the right side of equality.*

6 of the 27 preservice teachers who defined the equation as expressions with equality used the concept of algebraic expression in their definitions. 4 preservice teachers defined equation as equality of algebraic expression to a number and 2 pre-service teachers defined equation as equality of two algebraic expression.

"The equation is equality. It is equality that an algebraic expression is equal to a known value." 4x + 5 = 9 (PT27)

"It is the equality of two algebraic expressions." 5x + 48 = 12 (PT35)

In the interviews, it was seen that preservice teachers who defined the equation as the equality of two algebraic expressions, wrote examples, which is not congruent to their definitions. A part of the interview with Participant 35, which shows that the definition of the equation in the minds of preservice teachers is not clear, is presented as an example.

Researcher: *You defined the equation as the equality of two algebraic expressions. You wrote the expression, 5x + 48 = 12, as an example of equation. Can you explain?*

PT35: *There can be an unknown on the right side or equality or it can contain no unknown. It is important to have equality.*

Researcher: *Is 1 + 2 = 3 + 0 an equation?*

PT35: *Of course, it is an equation.*

27 of preservice teachers who defined the equation as expressions with equality, 4 used only the concept of equality in their definitions. Examples of preservice teachers' definitions are presented below.

"They are expressions that are equal to each other." 2 + 3 = 1 + 4 (PT23)

"It is the equality of two different expressions." 3x-5 = 10 (PT26)

2 of preservice teachers gave correct examples for the equation. In interviews conducted with these preservice teachers, they stated that the equation contains unknown. This shows that preservice teachers' definition of equations is missing. The other 2 preservice teachers wrote down number sentences as examples of equation. It is clarified in the interview that these preservice teachers thought that it was sufficient to have equality for the equation. The dialog below reflects this situation.

Researcher: *You have explained that the equation shows expressions that are equal to each other. You gave 2 + 3 = 1 + 4 as example. Can you explain?*

PT: *There is something known and proven. In my opinion, the equation is equality.*

Researcher: *Is 2x = 6 an equation?*

PT: *Yes, it has equality. If there is equality, it is the equation.*

Researcher: *Is it sufficient to have equality to be an equation?*

PT: *Yes.*

Expressions with unknown: 8 preservice teachers defined equation as "expressions that contain unknown". Although these preservice teachers did not use the concept of equality in their definitions, examples they gave contain equality and they gave the correct examples for the equation. In interviews conducted with 2 preservice teachers, they stated that the equation contains equality and the unknown is found thanks to equality. Examples of preservice teachers' definitions are presented below.

"Expressions that contain at least one unknown." 2x + 3 = 2y (PT24)

"Expressions with one or more unknowns." 2x + 13 = 47 (PT21)

Discussion and Conclusion

Mathematical definitions have an important role in mathematics teaching process. Defining mathematical concepts correctly is important for development of conceptual understanding (Edwards & Ward, 2004; Morgan, 2005; Mosvold & Fauskanger, 2013). Conceptual understanding is important for students' learning as well as for defining teachers' content knowledge competence (Morgan, 2005). Teachers' content knowledge is a determining factor of their teaching practices (Ball et al.,, 2008). Thus, it is very important to define basic algebra concepts correctly during algebra teaching process. In this context, this research focuses on how middle school mathematics teacher candidates define the concept of algebraic expression and equation as a dimension of content knowledge.

In the middle school mathematics curriculum (MEB, 2018), algebraic expression is defined as "expressions that contain at least one unknown and one operation". The algebraic expression definitions of the participating 35 preservice teachers classified under three themes: expression containing unknown (28 preservice teachers), expression containing equality (4 preservice teachers) and mathematical expression (3 preservice teachers). It is seen that definitions of preservice teachers (28 preservice teachers) who define the algebraic expression as expressions that contain unknown are insufficient. Most of these preservice teachers (12 preservice teachers) used only expressions containing unknown statements in their definitions. Chalouh and Herscovics (1988) stated that algebraic expression is often defined as expressions that involve variables. It was emphasized that such formal definition is not sufficient and meaningful for students to understand the concept. Defining algebraic expression only to a limited extent (defining algebraic expressions limited to variables) can make it difficult for students to understand other mathematical concepts (coefficient, term, constant term, and similar term) (MEB, 2018) contained in algebraic expression. Failure to understand these concepts correctly by students will make it difficult for them to learn advanced algebra topics (Kieran, Pang, Schifter & Ng, 2016; Stephens, Ellis, Blanton & Brizuela, 2017; Tirosh, Even & Robinson, 1988). For example, it is necessary to understand terms, coefficients and similar term concepts in order to learn the process of operations with algebraic expressions. For this reason, it is important to correctly express these concepts when defining algebraic expression. On the other hand, defining algebraic expression only as expressions containing unknown may cause students to develop different misconceptions that other mathematical concepts, which contain unknown, are also algebraic expressions.

Participants who defined algebraic expression as expressions containing unknown stated that algebraic expression also contains numbers and operations in their definitions (9 preservice teachers). This definition coincides with the definition given in the middle school mathematics curriculum (MEB, 2018). These preservice teachers gave correct examples for algebraic expression. 7 preservice teachers stated that algebraic expression does not involve equality. The reason why the preservice teachers emphasized that algebraic expression does not involve equality is due to their understanding that relation between algebraic expression and equation is operation-oriented. Interview data also support this situation. Two preservice teachers stated that there was no equality in algebraic expression, but it turns out to an equation when equality was added to algebraic expression. These explanations showed that preservice teachers explained the difference between algebraic expression and equation as operation-oriented (there is no equality in algebraic expression). They did not make any conceptual explanations regarding the mathematical meaning of why algebraic expression does not involve equality. Researches also show that preservice teachers explain basic algebra concepts with a focus on letters and symbols and focus on the operational process (Attorps, 2005; Black, 2007; Welder & Simonsen, 2011). Understanding the underlying mathematical meaning of operations is important for meaningful learning (Van de Walle, Karp, Bay-Williams, 2013).

4 pre-service teachers incorrectly defined algebraic expression as expressions with equality. These preservice teachers mentioned only the concept of equality in their definitions. 2 teacher candidates wrote down equation (e.g. $3x = 7$) and two teacher candidates wrote down number sentence (eg $2 + 3 = 5$) as an example of algebraic expression. 3 preservice teachers defined algebraic expression as mathematical expressions. These definitions are not an explanatory definition and examples given by the preservice teachers for algebraic expression (7/3, $\sqrt{5}$ and $\sin 30 = 1/2$) are incorrect. Based on these findings, it is understood that preservice teachers have wrong knowledge about algebraic expression.

In the middle school mathematics curriculum (MEB, 2018), the equation is defined as "equality that involve unknown and true for some values of the unknown." The majority of participants (28 preservice teachers) defined the equation as expressions with equality. 17 of these pre-service teachers defined equation as equality, which contain unknown. Preservice teachers gave correct examples for the equation. On the other hand, the teacher candidates' definitions do not exactly match up to the definition given in the middle school mathematics curriculum (MEB, 2018). In addition, since not every expression (eg identity) containing equality and unknown is an equation, preservice teachers' definitions are incorrect. These findings show that pre-service teachers define the concept of equation with a focus on letters and symbols. In the interviews, teacher candidates could

not elaborate their definitions. They could not make conceptual explanations about the meaning of the equation concept. The examples given by preservice teachers for the equation also support these findings. Preservice teachers think that the right side of equal sign is result while right side is the problem (Van de Walle, Karp, Bay-Williams, 2013). For this reason, they gave examples of equations where the unknown is on the left side of the equal sign. This shows that pre-service teachers consider the equation as computing the amounts and do not have knowledge about the relational meaning of equality.

4 pre-service teachers defined equation as the equality of algebraic expression to a number and 2 pre-service teachers defined as the equality of two algebraic expressions. The definitions of teacher candidates do not match the definition of the equation (MEB, 2018). These definitions do not meet the mathematical meaning of the equation concept. This shows that pre-service teachers think that the difference between equation and algebraic expression is existence of equality. The statements of prospective teachers that there is no equality in algebraic expression support this situation. 4 pre-service teachers defined the equation, using only the concept of equality, as equality of expressions. 8 of preservice teachers defined the equation incorrectly as expressions with unknown. These findings show that pre-service teachers focus on operational processes while defining the equation. Similar findings were obtained in other studies in the literature (Attorps, 2003; Tanisli & Köse, 2013; Yıldız, 2016). In the study carried out by Yıldız (2016) with three secondary school mathematics teachers, teachers defined the concepts of algebraic expression and equation with a focus on letters, operations and equality. They stated that the difference of the equation from the algebraic expression is the equation contains equal sign.

Recommendations

The results obtained from this research show that majority of participating preservice middle school mathematics teachers defines algebraic expression and equation wrongly. An important result of the study is that preservice teachers define algebraic expression and equation based on letters, operations and symbols. Similarly, teacher candidates did not make any explanations about the mathematical meaning of the concepts in the interviews and they focused on the operational processes. Similar results were obtained in the studies conducted in the literature (Attorps, 2003; Stephens, 2008; Strumps & Bishops, 2002; Yıldız, 2016). Ball (1990) pointed out that teachers should know the underlying meanings of operational processes, and that concept knowledge is an important part of teachers' content knowledge competencies. These results indicate that preservice education program should include applications that will enable preservice teachers to learn the definition and meaning of mathematical concepts. In the mathematics teaching courses, a teaching environment should be created in which preservice teachers can examine and discuss mathematical concepts not only theoretically but also practically.

In this study, the data were obtained from the written answers given by the preservice teachers to the questionnaire. One-to-one interviews were held with only some of the teacher candidates. In future studies, more in-depth findings can be obtained by conducting extensive interviews with fewer participants and using different data collection tools (focus-group interviews, observation of practice and internship courses, etc.).

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